THE COFFER OF THE GREAT PYRAMID.

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In an interesting article on the "Ancient Standards of Measure," which appeared in the *Quarterly Statement* for July, 1899, General Sir C. Warren discusses, among other matters, the method by which the dimensions of the sarcophagus, or coffer, in the Great Pyramid were arrived at (*see* pp. 252 to 257). He gives some rather elaborate calculations in order to obtain the measures probably intended by the original designer, and works these out to several decimals of a British inch.

It appears to me that his conclusions are much more complicated than is necessary, and that it is rather more likely that the vessel was constructed on the ordinary measures in use in Egypt. A study of the coffer certainly leads to this, and indicates a much simpler, though at the same time a very ingenious, design.

The ancient scales which have been found, and the measurements of the monuments, prove that the usual building unit in ancient Egypt was a cubit of about 20·65 British inches in length, divided into 7 palms or handbreadths, each about 2·95 inches in length. This cubit is frequently called "the royal cubit," to distinguish it from the common cubit, which was 6 palms in length. Some writers are doubtful whether the measure of a common cubit was used in Egypt, but it cannot be questioned that a cubit of 6 palms is much more nearly equal to the ordinary cubit of a man than a cubit of 7 palms, and the suggestions which I propose to bring forward with regard to the coffer of the Great Pyramid certainly lead to the conclusion that this smaller cubit was regarded as a measure by the Pyramid builders.

This suggestion is that the dimensions of the coffer—lineal, superficial, and solid—are based, not upon the length of the cubit of 7 palms only, but upon a combination of the lengths of this cubit with that of the common cubit of 6 palms.

It is necessary in the first place to show that these dimensions can be expressed in terms of the Egyptian palm of 2·95 British inches. Dr. Petrie, whose measurements of the Pyramids were taken with an accuracy and care worthy of the greatest admiration, devoted special attention to the determination of the dimensions of the coffer, and he took no less than 950 different measures of it, so as to arrive at an accurate mean result. He has pointed out that the coffer was by no means perfectly made, and the rough usage to which it has been exposed for a great number of years has no doubt appreciably altered it in some respects. At the same time we cannot be wrong in taking the mean dimensions which he has recorded as not being far from the original dimensions of the vessel, as it existed when placed in the Pyramid. These mean dimensions are as follows :—

Mean exterior length	.. ..	89·62	British inches.
,, depth	.. ..	41·31	,, ,,
,, breadth	.. ..	38·50	,, ,,
Mean interior length	.. ..	78·06	,, ,,
,, depth	.. ..	34·42	,, ,,
,, breadth	.. ..	26·81	,, ,,
Mean thickness of north end	..	5·67	,, ,,
,, ,, east side	..	5·87	,, ,,
,, ,, south end	..	5·89	,, ,,
,, ,, west side	..	5·82	,, ,,
,, bottom ..	..	6·89	,, ,,

If we divide each of these numbers by 2·95, we obtain the numbers of Egyptian palms which are most nearly contained in each of the mean dimensions, as recorded by Dr. Petrie. The following table shows the results as thus calculated :—

—	Calculated Dimensions. In palms of 2·95 British Inches.	Calculated Dimensions. In British Inches.	Actual mean dimension as given by Dr. Petrie, in inches.	Difference in Inches.
Exterior Length ..	30	88·50	89·62	+1·12
,, Depth ..	14	41·30	41·31	+0·01
,, Breadth ..	13	38·35	38·50	+0·15
Interior Length ..	26	76·70	78·06	+1·36
,, Depth ..	$11\frac{2}{3}$	34·42	34·42	0·00
,, Breadth ..	9	26·55	26·81	+0·26
Thickness, North End	2	5·90	5·67	−0·23
,, East Side	2	5·90	5·87	−0·03
,, South End	2	5·90	5·89	−0·01
,, West Side	2	5·90	5·82	−0·08
,, Bottom	$2\frac{1}{3}$	6·88	6·89	+0·01

It will be seen that the calculated dimensions in palm agrees very closely with the actual mean dimensions. The only error of any importance is that the coffer is about an inch longer than it should have been, and this is only a little over 1 per cent. of the length. Probably this error was not noticed when the block was cut out of the quarry. The same small excess is naturally repeated in the length of the interior, as the line for cutting out the latter must have been set off by marking a width of two palms all round the top exterior edge. The error was slightly increased by cutting the north end of the coffer about a quarter of an inch too thin.

As Dr. Petrie has pointed out, and as anyone can see by examining the coffer, it was not finished with mathematical accuracy, and the small differences between the theoretical and actual dimensions are less than might have been expected.

Having thus arrived at the dimensions probably intended in palms, I will show how they can be converted into cubits.

The exterior length is 30 palms, which are equal to 5 common cubits of 6 palms each.

The exterior depth is 14 palms, which are equal to 2 royal cubits of 7 palms each.

The exterior breadth is 13 palms, which are equal to 1 royal and 1 common cubit.

The interior length is 26 palms, which are equal to 2 royal and 2 common cubits.

The interior depth is $11\frac{2}{3}$ palms, which are equal to $1\frac{2}{3}$ royal cubits.

The interior breadth is 9 palms, which are equal to $1\frac{1}{2}$ common cubits.

Perhaps it will be simpler to use the letter R to express the length of a royal cubit, and C to represent the length of a common cubit. With these symbols the dimensions may be expressed as follows:—

Exterior length	=	5 C.
,, depth	=	2 R.
,, breadth	=	R + C.
Interior length	=	2 (R + C).
,, depth	=	$1\frac{2}{3}$ R.
,, breadth	=	$1\frac{1}{2}$ C.
Thickness of sides and ends	=	$\frac{1}{3}$ C.
,, bottom	=	$\frac{1}{3}$ R.

The whole of the linear dimensions can, therefore, be simply expressed in terms of the lengths of the royal and common cubit used in combination.

The superficial dimensions can be calculated without difficulty. The exterior surface of the coffer is equal to the sum of the surface of the four sides and the bottom added to the surface of the top, of two palms in width all round the coffer.

Taking the dimensions in palms, the exterior surface is equal to $86 \times 14 + 30 \times 13 + 78 \times 2$, equal to 1,750 square palms. As a square royal cubit is equal to 49 square palms, and a square common cubit is 36 square palms, it is easy to see that the exterior surface $= 10\ R^2 + 35\ C^2$, using R and C, as before, to denote the lengths of the royal and common cubit.

The interior surface is equal to the sum of the four sides and bottom, and, taking the dimensions in palms, is equal to $70 \times 11\frac{2}{3} + 26 \times 9$, equal to $1{,}050\frac{2}{3}$ square palms. This reduced to cubits is equal to $6\ (1\frac{2}{3}\ R)^2 + 3\ C\ (R + C)$. If the $\frac{2}{3}$ square palm is neglected, and the interior surface is taken as equal to 1,050 palms, it can be expressed by the form $6\ R^2 + 21\ C^2$, or just three-fifths of the exterior surface.

I will now consider the exterior and interior solid dimensions, which are the most interesting. The total exterior capacity is equal to $30 \times 14 \times 13 = 5{,}460$ cubic palms, and the interior to $26 \times 9 \times 11\frac{2}{3} = 2{,}760$ cubic palms, so the exterior appears to have been designed to be double the interior capacity. The volume of stone in the coffer is, of course, equal to the interior capacity; at least, this appears to have been the intention of the designer, although it can be seen from Dr. Petrie's calculations that the proportion of the volume of stone to the interior capacity is as 1 to 1·018, instead of 1 to 1. This is evidently due to the fact that the sides of the interior were cut away rather too much, as I have already shown on p. 153.

The exterior capacity can be conveniently represented in terms of the royal and common cubits by the following formula. Capacity $= 10\ R\ C\ (R + C)$. This is a neat combination of the two cubits.

The interior capacity $= 5\ R\ C\ (R + C)$.

There is another way of representing the exterior capacity which, if intended, was very ingenious. It is almost exactly equal to the sum of the volumes of a cube of 2 royal cubits' side

(14 palms), and of a cylinder of 2 common cubits in radius and 1 common cubit in height.

		Cubic palms.
Cube of 14 palms	=	2,744
Cylinder of 12 palms radius and 6 palms in height $\left(\text{taking } \pi \text{ as equal to } \frac{22}{7}\right)$	=	2,715·43
Total		5,459·43

5,459·43 is almost exactly equal to 5,760, which I have already shown to have been the probable designed capacity of the coffer. A very small alteration in the value of π would make the comparison exact. This value for the external capacity is expressed by the formula—

$$\text{Exterior capacity} = (2\,R)^3 + 4\,\pi\,C^3.$$

The interior capacity, being half the exterior, is of course equal to the mean of the volumes of a cube of 2 royal cubits side, and of a cylinder of 2 common cubits radius and 1 common cubit in height.

The volume of the stone in the sides and bottom of the coffer appear also to follow a simple rule.

As the thickness of the sides and ends appears to have been intended to be 2 palms, and the thickness of the bottom $2\frac{1}{3}$ palms, we have the relations:—

Volume of bottom	$= 30 \times 13 \times 2\frac{1}{3}$	$= 910$	cubic palms.
,, one side	$= 26 \times 11\frac{2}{3} \times 2$	$= \frac{2}{3}(910)$	,,
,, one end	$= 13 \times 11\frac{2}{3} \times 2$	$= \frac{1}{3}(910)$	,,

The volume of the bottom is therefore equal to the sum of the volume of one side and one end, and is also equal to one-third of the interior capacity of the coffer.

I have no idea why the two cubits should have been used in the construction of the coffer, but it is rather remarkable that the use of a long and short cubit in the same structure appears also to have been adopted in the great altar of sacrifice in the Temple of Jerusalem, for we read in the Beth Habbechereh, chap. i, 6,[1] that "of the 10 cubits in the height of the altar some were of 5 handbreadths and some were of 6 handbreadths."

[1] See *Quarterly Statement*, 1885, p. 39.

The Babylonian cubit of 5 handbreadths corresponded to the Egyptian cubit of 6 handbreadths, and that of 6 handbreadths to the Egyptian of 7 handbreadths.

The use of the two handbreadths also appears in the dimensions of the king's chamber of the Great Pyramid, in which the coffer is placed.

Taking again Dr. Petrie's measurements, we get the following dimensions in palms for the king's chamber :—

—	Calculated Dimensions.		Actual mean dimension as given by Dr. Petrie.	Difference in Inches.
	In palms of 2·95 British Inches.	In British Inches.		
Length	140	413·00	412·65	−0·35
Breadth	70	206·50	206·29	−0·21
Height	78	230·10	230·10	0·00

Using the same symbols as before—

The length.. ..	=	20 R.
Breadth	=	10 R.
Height	=	6 (R + C).
Content	=	1,200 R^2 (R + C).
	=	1,400 R C (R + C).

The content of the king's chamber is therefore equal to 140 times the capacity of the coffer.

On the whole, I think that the explanation of the dimensions of the coffer by the combination of the two cubits appears to be simpler than that given by Sir C. Warren. It may probably have some hidden meaning, but as to this I do not venture to give any opinion.