

TRIANGULAR NUMBERS IN THE NEW TESTAMENT.

IN a recent number of the JOURNAL Dr McNeile called attention to an article by Dr van den Bergh van Eysinga in the *Zeitschrift für die neutestamentliche Wissenschaft*, in which a new explanation was suggested for the number of the Beast. In this article it was pointed out that $666 = 1 + 2 + 3 \dots + 36$, while $36 \text{ again} = 1 + 2 + 3 \dots + 8$. Evidence was then brought from Philo and elsewhere to shew that these 'triangular' numbers ($= 1 + 2 + 3 \dots + n$)¹ were held to have the symbolic value of n . Thence the conclusion was drawn that 666 had the symbolic value of 8, which in Gnostic symbolism = wisdom. Thus we get a new meaning at any rate for the words 'here is wisdom'.

Dr McNeile did not however mention, what to me seemed the most significant point in Dr van Eysinga's paper, viz. that the other well-known Johannine number 153 is also triangular, being equal to $1 + 2 + 3 \dots + 17$. This coincidence seemed to me to give a *prima facie* ground for looking further into this arithmetical theory. The materials for what I have to say about it will be found in Gow's *History of Greek Mathematics*, and in the short treatises of Nicomachus of Gerasa and Theon of Smyrna, both of which are edited in the Teubner series.

The doctrine of 'triangular numbers' seems to belong to the earlier form of Pythagorean arithmetic. The meaning of the name is clearly explained by both the writers mentioned above with diagrams. Arrange your units (symbolized by a) in parallel lines as follows :—

$$\begin{array}{l}
 (1) \quad a \\
 (2) \quad a \quad a = 3 \\
 (3) \quad a \quad a \quad a = 6 \\
 (4) \quad a \quad a \quad a \quad a = 10
 \end{array}$$

and so on. The whole number in each case, 3, 6, 10, &c., will form an equilateral triangle, with the side (technically called the gnomon) equal to the last number of the series. Thus 153 fishes will be conceived of

¹ May I explain for the benefit of those who have forgotten their elementary algebra, that the sum of $1 + 2 + 3 \dots + n = n \cdot \frac{n+1}{2}$, and that therefore a triangular number is half the product of two consecutive numbers, e. g. $666 = \frac{36 \cdot 37}{2}$.

as forming an equilateral triangle with a side of 17. It is thus easy to see why any symbolic value that may attach to 17 will also attach to 153.

As I have said the theory of triangular numbers is early, but with other kindred ideas, two of which I shall mention later, it appears to have been popularized very much about the time that the New Testament or at least its later books were written. Arithmetic, says Gow, disappears from history about 100 B.C., but was revived by Nicomachus of Gerasa, from whose time it became the favourite branch of mathematics. Nicomachus became extremely popular. He was translated into Latin in the second century by Apuleius and by Boethius in the sixth, and was the subject of some Arabian commentaries. In Lucian's time he was the type of the arithmetician or calculator.¹ He was succeeded by Theon of Smyrna, who was most probably living and working at A. D. 140. Both these men write avowedly in the interests of philosophy, of which they regard arithmetic as the vestibule. Nicomachus was a Pythagorean and Theon a Platonist.

The date and home (or possibly birthplace) of Nicomachus himself is a matter of some interest in our present enquiry. He came probably from Gerasa in the Decapolis, about twenty miles from the Jordan. This date is given by Gow as about A. D. 100, but the evidence is consistent with some decades earlier. Liddell and Scott in fact give his *floruit* at A. D. 50. If so, he is both a contemporary and a near neighbour of the Apostles.

In both writers closely bound up with the 'triangulars' are two other systems. The first is the *τετράγωνοι* or square numbers. These are the same as our square numbers, but they are conceived of as generated in the following way. Start as in the triangulars with one unit *a*; then add a 'gnomon' of 3, making the square number 4, then a 'gnomon' of 5, making the square number 9, then of 7 making 16, and so on.

<i>a</i>	<i>a a</i>	<i>a a a</i>	<i>a a a a</i>
	<i>a a</i>	<i>a a a</i>	<i>a a a a</i>
		<i>a a a</i>	<i>a a a a</i>
			<i>a a a a</i>

Thus the *τετράγωνοι* are conceived of as the sum of successive *odd* numbers.

Again, closely connected with these are the *έτερομήκεις*.² No diagram

¹ *ἀριθμητής ὡς Νικόμαχος ὁ Γερασσηνός (Philopatris 12).*

² I do not know how to translate this word. Not 'oblong', which is *προμήκης*. Boethius gives 'altera parte longiores'.

is given, but it can easily be supplied. Start with *two* units : then add 4, making 6, then add 6, making 12, and so on.

a	a	a	a	a	a	a	a
		a	a	a	a	a	a
			a	a	a	a	a
				a	a	a	a

Thus the *ἑτερομήκης* will always form a rectangle of which one side is shorter by one unit than the other. It is therefore the product of two consecutive numbers, and is of course always double of some triangular number. It is also the sum of successive *even* numbers. It will thus be seen that these three systems form a compact whole.

(1) *τρίγωνοι* (sum of successive numbers).

$$1 + 2 + 3 \dots + n = n \cdot \frac{n + 1}{2}.$$

(2) *τετράγωνοι* (sum of successive odd numbers).

$$1 + 3 + 5 \dots + n = \left(\frac{n + 1}{2}\right)^2.$$

(3) *ἑτερομήκεις* (sum of successive even numbers).

$$2 + 4 + 6 \dots + n = \frac{n}{2} \cdot \frac{n + 2}{2}.$$

A passage from Nicomachus in which he describes the relations of these last two systems is worth quoting, as shewing the romance and interest which attach to these numbers. He arranges the first few of the *τετράγωνοι* and *ἑτερομήκεις* in parallel lines as follows :—

	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
<i>τετράγωνοι</i>	1	4	9	16	25	36	49	64	81	100
<i>ἑτερομήκεις</i>	2	6	12	20	30	42	56	72	90	110

He then shews that to form an *ἑτερομήκης* for the corresponding *τετράγωνος*, you must add the number which represents its place in the series : e.g. 64 being the 8th *τετράγωνος*, you must add 8 to make the 8th *ἑτερομήκης* viz. 72. He then proceeds.

‘When we contemplate (*ἐνατενίζοντες*) them as set forth we shall marvel at their mutual harmony (*φιλαλληλία*), and how they help each other to generate and perfect those that succeed, and thus naturally suppose that in universal nature a similar result is accomplished by the providence which orders the Cosmos (*κοσμική πρόνοια*).’

The suggestion I now have to offer is that this Nicomachean arithmetic, if I may so call it, has distinctly influenced the numbers of some books of the New Testament. And first let us take the Apocalypse. It will not be questioned that (1) the writer had a very great interest in numbers, (2) that his choice is largely based on Old Testament usage. But is there another element ?

The smaller numbers cannot be said to shew anything one way or another. His favourites are 4, 7, 10, 12. It is true that of these 4 is square, 10 triangular, and 12 *ἑτερομήκης*, but no argument can of course be based on this. In the first place they are all Old Testament numbers, in the second place the numbers belonging to the three systems occur so frequently among early numbers, that nothing can be inferred from their appearance.¹ All we can say is, that so far we have nothing incompatible with a knowledge of Nicomachean arithmetic. But when we rise to the higher numbers we get a more suggestive result. Here we must naturally disregard the thousands. No one, however addicted to the Nicomachean series, can be expected to dispense with the use of thousands to represent large multitudes. Apart from these we have—

- (a) The number of the Beast 666 (xiii 18).
- (b) 1,260 days (xi 3 and xii 6).
- (c) The blood pours out of the winepress for 1,600 stades (xiv 20).
- (d) The wall of the New Jerusalem is 144 cubits high (xxi 17).

Of these (a) van Eysinga has shewn that 666 is the 'triangle' of 36, but one point, which he has not observed, may be mentioned. In Plutarch (*de Is. et Os.* 76) we are told that 36 was held peculiarly sacred by the Pythagoreans, and that it was called the great tetractys, because it was the sum of the first four odd and the first four even numbers. This is of course merely a way of stating that it is equal to $1 + 2 + 3 \dots + 8$.² It should be observed that elsewhere the name of the 'great tetractys' seems to be reserved for 10 which = $1 + 2 + 3 + 4$. 36, says Plutarch, was their greatest oath, and signified the Cosmos. This last statement seems rather suggestive in connexion with the Beast. We have, of course, no specific evidence that the honour given to 36 was extended to 666, but it would naturally follow from the general principle of triangular numbers. When we take into account the identity of what we call the digits in 666,³ it seems likely that in Pythagorean or Nico-

¹ It may help readers to estimate the value of the argument, if I subjoin the following: The total proportion of numbers belonging to the three systems is between

1 and 50—44 per cent.
50 and 100—18 per cent.
100 and 1,000—8 per cent.
1,000 and 2,000—4½ per cent.

² Zeller suggests that 36 was also sacred as being the sum of the first three cubes = $1 + 8 + 27$. I do not know whether there is any definite evidence that this was recognized. It also possesses the rare, if not unique, property of being both square and triangular.

³ This of course is not so striking in the Greek numerical system. But the words *ἑξακόσιοι*, *ἑξήκοντα*, *ἕξ* would by their similarity strike the imagination. In fact this is the one point about the number which does strike Irenaeus.

machean circles 666 would be a very king amongst numbers. I will leave for the present the question of the variant 616.

(*b*) The 1,260 days which occur in two connexions are usually assumed to be another way of expressing 42 months (xi 2 and xiii 5), and 'a time, times, and half a time (xii 14) or $3\frac{1}{2}$ years'. This may be so, though one may doubt whether the seer would without some special reason have made the year consist of only 360 days. Anyhow the fact remains that 1,260 is $\epsilon\tau\epsilon\rho\omicron\mu\acute{\eta}\kappa\eta\varsigma = 35 \cdot 36$, as also is $42 = 6 \cdot 7$.

(*c*) and (*d*) 144 and 1,600 are of course both square numbers. They belong to the Old Testament scheme of numbers, in so far as their 'roots' are typical Old Testament numbers. But as they represent lengths not areas, there is no apparent reason for the squaring.

Two other Apocalyptic numbers may be noted.

(*a*) Amongst the thousands 144,000 is the most striking. It is not of course properly speaking a square number, and it is attained by the ordinary process of multiplication. Given 12,000 per tribe it follows necessarily. Still it harmonizes with the seer's love of square numbers.

(*b*) The number of the 24¹ elders has given rise to a good deal of speculation. It is not an Old Testament number, and it does not belong to our three systems. But it may be observed that the 24 elders are always (I do not think xi 16 is really an exception) coupled with the 4 living creatures. That is to say there are always 28 round the Throne.

And 28 is of course triangular = $\frac{7 \cdot 8}{2}$ ². This can no doubt be brought into the argument, only on the assumption, that when a writer mentions two closely connected and juxtaposed numbers, he wishes to bring their sum (though unexpressed) before our minds. Such an assumption would, I think, be made by many readers in one familiar case. Many probably feel that in the miracle of the 5,000, the Evangelists in speaking of 5 barley-loaves and 2 small fishes wish to direct our minds to the mystic number 7. Still it is of course conjectural.

Leaving out these two doubtful numbers, we have the fact that all the five largest numbers of the Apocalypse (exclusive of thousands) belong to the three systems. I find it difficult to resist the belief, that

¹ I suspect that one at any rate of the associations present in the seer's mind, when he assigned the number 24 to the Elders, was the fact that the Greek alphabet has 24 letters. That the alphabet had a romance for him, which it has not for us, is obvious. Which of us could describe the deity as A and Z? The chapter on the alphabet in Dionysius Thrax (for centuries the accepted grammar book) opens with the words, 'There are 24 letters from Alpha to Omega'. The name $\sigma\tau\omicron\chi\epsilon\acute{\iota}\alpha$ was in itself suggestive. An interesting illustration of this will be found in Mr Gaselee's *Parerga Coptica* i.

² It is also in Pythagorean arithmetic a 'perfect' number, as being the sum of its factors $1 + 2 + 4 + 7 + 14 = 28$.

St. John though working on an Old Testament basis is influenced by Nicomachean arithmetic.

When we turn to the narrative books we find in the Fourth Gospel and the Lucan writings phenomena, which though in themselves probably unconvincing, become significant, if taken with the facts of the Apocalypse. In these books three numbers stand in a category by themselves, as what I may call large definite statistical numbers, numbers that is which the reader would naturally regard as the result of a pretty careful counting of a collection of people or things assembled by chance. These are

153 fishes (John xxi);

(about) 120 at the election of Matthias and presumably at Pentecost (Acts i);

¹ 276 in St Paul's ship (Acts xxvii).

All these three are triangular. I have already noted that

$$153 = \frac{17 \cdot 18}{2} = 1 + 2 + 3 \dots + 17.$$

$$120 = \frac{15 \cdot 16}{2} = 1 + 2 + 3 \dots + 15, \text{ and } 15 \text{ again} = 1 + 2 \dots + 5, \text{ so}$$

that this number like 666 is doubly triangular.

$$276 = \frac{23 \cdot 24}{2} = 1 + 2 + 3 \dots + 23.$$

All three are connected with some supernatural event or revelation. For the 276 of Acts xxvii is not a mere official statistic. When all hope of safety had been lost, the Angel of the Lord had stood by Paul and assured him that God had granted him all that sailed with him. This assurance was twice conveyed by Paul to the crew, and was verified by the sequel contrary to all human probability. It is in connexion with these facts that St Luke says 'now we were in all in the ship 276 souls'.

It may be noted that the triangular character of the 120 and 153 were known to the fathers. Jerome (Ep. 53)² remarks that the number at Pentecost 'rising gradually and by increments from 1 to 15 makes the number of 15 degrees which are mystically contained in the Psalter'. So, too, Augustine in his tractate on John xxi shews by addition that 153 is the 'triangle' of 17.³

¹ It will be remembered that B (supported by the Sahidic) has 'about 76'. But here no doubt *πλοῖψ σοϚ* had been corrupted to *πλοῖψ ως οϚ*.

² 'Qui centum viginti ab uno usque ad quindecim paulatim et per incrementa surgentes, quindecim graduum numerum efficiunt, qui in Psalterio mystice continentur.'

³ 'Qui numerus (i. e. septemdecim) ab uno usque ad seipsum computatis omnibus crescens ad centum quinquaginta tres pervenit.' I owe this reference to Mr W. Montgomery.

If we examine the other numbers in these books we find (1) the familiar groupings of the disciples 7, 12, 70, and the 40 days, all of which are typical Old Testament numbers; (2) various notes as to distances, sums of money, chronological data, ages, or periods in human life, and the like. Most of these are either small or round numbers.¹ Even here, however, one or two odd possibilities crop up. The highest number amongst the age records, that of Anna, on the principle suggested above, that two juxtaposed numbers may be consciously added together gives the triangular number 91 ($= \frac{13 \cdot 14}{2}$). Again, in Luke x 1 the very well supported variant 72 is *ἑβδομήκην* (= 8.9). These are, of course, merely possibilities and only strengthen the argument, in so far as they weaken the counter-argument, which may be drawn from the frequency of numbers, which do not belong to the three systems.

The question now is whether the facts, which I have brought forward, taken all together fall within the reasonable limits of coincidence. I hardly think that they do, and I would ask readers who think otherwise, to let me put before them a little sum in probabilities. There are in these books, and indeed I think in the whole of the New Testament,² only seven of what I should call definite numbers above a hundred (120, 144, 153, 276, 666, 1,260, 1,600). Now the numbers between 100 and 1,600, which belong to our three classes, are 103, or about 1 in 14 or 15. The odds therefore against all seven belonging to these three classes are somewhere about 120 millions to 1. I daresay some exception may be taken to this way of stating it, but it will require a vast amount of exception to reduce the odds to a point where coincidence becomes probable. Still, unless some general principle can be found, which will account for the predilection shewn for such numbers by the authors of the Fourth Gospel, Acts, and Revelation, I can well understand that many will still prefer to regard it as a coincidence.

Such a principle can hardly be found in symbolism. The numbers do not seem to lend themselves to any such treatment, with the exception perhaps of 666. On the symbolic principle $120 = 15$, $153 = 17$, $276 = 23$, but what are we to make of 15 and 17 and 23? We have seen indeed that Jerome brings in the 15 'degrees' of the Temple. Augustine explains 17 as an addition of 10, symbolizing

¹ 46 in John ii 20 (apparently a careful piece of chronology) and 38 in John v 5 are rather marked exceptions.

² The only number answering to this description, which I have observed, is the 430 years of Gal. iii 17, which as a chronological datum derived direct from the Old Testament may be fairly said not to count.

the law, to 7 symbolizing the Gospel. Van Eysinga thinks that the author of the Gospel intended to indicate the nations represented according to Acts ii at Pentecost (which will add up to 17). It seems to me that the modern critic outdoes the ancient allegorists in fancifulness. Moreover, I doubt on my present knowledge, whether symbolism plays a large part in the later Pythagorean theory of numbers. Certainly there is nothing of the sort in Nicomachus and Theon. Their relation to the original Pythagorean symbolism seems to me much the same as that of Wordsworth's view of nature to Keble's, as shewn in the hymn 'There is a book, who runs may read'. While to Keble the sky meant the Maker's love, the moon the Church, and the stars the saints, to Wordsworth all nature was a revelation of divine beauty. So with Nicomachus different numbers did not mean different things, but the endless mysteries and harmonies revealed in the properties of numbers were a sign and proof of 'Cosmic Providence'.

I would explain the New Testament phenomena by supposing that these numbers had become to certain minds a matter of profound interest. This feeling probably took a different shape in different minds. In some it may have been superstitious or at least mystical, in others it may have represented a genuine intellectual curiosity. Many readers may be inclined to think that in St John the first feeling was predominant, in St Luke the second. Such an interest might have a threefold result. Firstly, when the imagination is largely free as it presumably is in the Apocalypse, such numbers may be deliberately chosen. Secondly, when such numbers actually occurred, the circumstance is more likely to be recorded. Thirdly, there may be a tendency to imagine that a number is triangular when it is really something less interesting. I can illustrate such a tendency from a recent experience of my own. Forty years ago I remember a great cricket match at Haileybury, where the captain played a splendid innings of (as we thought) 153 not out. I remember a friend of mine exclaiming 'although they were so many yet was not the net broken', and all these years I should have affirmed most positively that the score on this occasion was 153. A few months ago I had occasion to look back at the old school magazines and found that it was really 152, and so recorded in the magazine of which I was at the time myself an editor. I can only suppose that the aptness of the quotation and the association obliterated from my mind the real facts.

If we apply these principles to the number of the Beast we need not come with van Eysinga to the conclusion that the old explanation of a 'gematria' is to be rejected. On the contrary, such a 'gematria' would be more acceptable, if it could be made to harmonize with a sovereign number. And further we may expect that the working of

the gematria might be somewhat strained to get this result. If Nero Caesar in Hebrew is otherwise suitable, we need not quarrel as van Eysinga does with the fact that it is Hebrew not Greek. It might be well worth while to use Hebrew letters, if thereby the writer secured a number with the properties of 666—perhaps we may add a number which bore the very appropriate meaning of the Cosmos—a term which to the Pythagorean would suggest the ordered universe, but by a Christian would be taken in the sense which it bears so often in the Fourth Gospel.¹ It should be added that the virtues of 666 make it all the more necessary to account for the variant 616. The facts seem to me to suggest strongly that 616 is not indeed the original reading in the text, but is the original form of the gematria which St John reshaped. And this is quite in accordance with the fact that *Ταῖος Καῖσαρ* does satisfy 616.

On the whole, the remarkable fact which van Eysinga has pointed out, and the other facts which I have added, seem to me to leave the riddle of the Beast much where it was. Have these facts then any real significance or value? When this paper was discussed at the Cambridge Theological Society, Professor Burkitt expressed an opinion that the residual truth which appeared was that we must enlarge our ideas of what were interesting numbers to the early Christian mind. I should put it a little higher than this. It seems to me a real link between the New Testament writers and the speculation and science of the time. If 'science' seem too lofty a word, it must be remembered that Greek arithmetic worked out its results under drawbacks which we hardly realize. In the first place, multiplication, as we know it, was a difficult process with Greek notation.² In the second place, the Greeks at this time had no algebra. Thus while Nicomachus infers from the geometrical form of his diagrams that the *ἑτερομήκεις* and *τετράγωνοι* were respectively of the form $r \cdot r + 1$ and r^2 , he shews no knowledge of the corresponding property of the *τρίγωνοι*, where his diagram did not help him. In fact the simple formula

$$1 + 2 + 3 \dots + n = n \cdot \frac{n + 1}{2},$$

which is now within the grasp of an ordinary schoolboy, seems to have been beyond his ken. The calculation therefore of high triangular numbers was worked by addition, and if St John really knew that 666 was the 'triangle' of 36 he knew a fact which could only be learnt with a great deal of patience and labour.³ It must be re-

¹ And in the Apocalypse itself (xi 15).

² v. Gow l. c. pp. 49-51.

³ This may perhaps account for the fact that, while the triangularity of 120 and 153 was observed by Jerome and Augustine, that of 666 is not known apparently by Irenaeus, or, so far as I know, by any of the Fathers.

membered that the Church was born in an age which gave great attention to education, and had in its own way a really scientific spirit. I would diffidently suggest that the Mystery religions and the Gnosticism to which so much attention is now given were but side-breaths of the time-spirit, and that even philosophy was not a universal study. The main current of intellectual life and thought followed the regular channels of education—grammar and literature, rhetoric and history, arithmetic and geometry ; and a closer study of these may conceivably throw a great deal of light on the thought and terminology of the New Testament and early Christian writers.

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