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USE AND MISUSE OF MATHEMATICS

By E. H. Betts, B.Sc.

THIS paper endeavours to answer two questions: What is mathematics? and, What is its logical status when applied to other sciences?

Historically, mathematics doubtless originated in practical transactions and land-surveying, requiring and leading to the use of number and counting and the measurement of distances and areas. Hence it has been traditionally defined as "the science of discrete and continuous quantity." But the almost incredible development of arithmetic and geometry with their widening generality and consequently increasing abstractness, their ever-extending use of symbols, their postulational, deductive—and hence "creative"—systems, has so transformed the subject that the old definition will no longer contain it. It is obvious, for example, that projective geometry, which deals with non-mensural facts and relations, would be excluded by the above definition, as would other vast tracts of modern mathematics. The closer scrutiny given in the nineteenth century to the logical foundations of the subject, including especially the definitions and axioms of Euclid's geometry, revealed the defective plausibility of the famous "parallel axiom," the reliance placed on spatial intuitions and the hitherto unsuspected use of many implicit assumptions. This brought into prominence the possibility of geometries other than Euclidean based on the denial of the parallel axiom. Such logically consistent deductive systems having been actually constructed, not only geometries but also algebras—quaternions, e.g.—it appeared that if account were taken of these and all other recent developments, the characteristic activity of mathematics must be held to be deduction. Accordingly, as long ago as 1870, mathematics was defined by the American mathematician Benjamin Peirce (*Linear Associative Algebra*) as "the science which draws necessary conclusions." This was one of the earliest recognitions that mathematics is not tied either to number (discrete magnitude) or to geometry (continuous magnitude) or indeed to any particular subject-matter, but that "it belongs to every inquiry, moral as well as physical " (*loc. cit.*). The layman may well ask what then is the distinction between mathematics and logic? The modern majority reply is that there is none. For the great effort of recent mathematical philosophers has been to reduce all the concepts and propositions of the former to those of the latter—and at the same time to insist on the severest rigour by dint of making every necessary assumption explicit. Whitehead, for instance, defines mathematics as " the science concerned with the logical deduction of consequences from the general premises of all reasoning."[1] Bertrand Russell's version, fully in accord with this, is that mathematics is " the class of all propositions of the form '*p* implies *q*' where *p* and *q* are propositions and neither *p* nor *q* contains any constants except logical constants."[2] The actual exhibition of mathematics as a construction built up solely on the notions and principles of pure logic has been given us in *Principia Mathematica*, the classic work of Russell and Whitehead—which has, however, by no means been found to be above criticism, and that on logical grounds. It will doubtless be argued that this identification of mathematics with formal logic will exclude the use, in mathematical work, of experimental methods, of the analogy of the mechanical model, of the *feeling* for pure form, of the intuitive flash and of the (often fruitful) fumbling towards a result—methods and processes which have admittedly been productive during the long history of mathematics of many, if not most, of its greatest discoveries. The use by mathematicians of these methods in actual discovery is not denied. But however arrived at, and whatever be the machinery of discovery, no proposition takes its place in the system of mathematical knowledge proper until it has been " proved," that is to say, logically deduced from previously established propositions. An inquiry into the nature of mathematics would not therefore be satisfactory without examination of the nature of those propositions which are logically prior to all others, and also of the ultimate notions the relations between which are stated in these logically prior propositions. Full discussion of this is beyond the scope of this paper. Some important conclusions may however be given. (i) The idea of logical priority

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[1] *Ency. Brit. XIV, Art. Maths. Nature of*, Vol. 15, p. 86.

[2] *Prins. of Maths.*, p. 3. A logical *constant* is a symbol which signifies logical form, e.g. *ent* (=" entails ") signifies the form of implication. " If . . . then . . ." *Ent* in any logical calculus is a constant. A *variable* is a symbol employed to give generality instead of any particular material constituent, e.g. *x* instead of *Caesar*, *a* instead of *generals*.

is relative and not absolute, i.e. it relates to priority within any given system and not to general priority. (ii) Any deductive system, and therefore mathematics, as such, must commence with certain *undemonstrated propositions* (" primitive propositions " as they were called by Peano) and certain *undefined elements* (" primitive concepts "). (iii) Since it must be an ultimate law or postulate of all thought that truth cannot contradict itself it is necessary for the primitive propositions to be mutually consistent. But (iv) They need not be demonstrably *true*, i.e. in accord with perceptual experience; for demonstration of truth in this sense belongs to physics or natural science. (v) It has also been shown by the work of Peano, Frege and Russell that for mathematics in general the primitive concepts are those of ordinary logic, e.g. a *class*, *belonging to a class*, and *similarity*, and not our intuitively acquired notions of space or of the natural numbers.[1] In short, mathematics, radically viewed, is the science of reasoning from given premises. Whatever its methods—use of symbols, e.g.—its processes are indistinguishable from those of ordinary inference. Even its symbols are available for use in ordinary reasoning where necessary. De Morgan showed, for instance, how difficult it often is to answer simple questions in the logic of relations without the use of symbols, and adduced as an example : " What people are not the descendants of those who are not my ancestors ? "[2] It is not, therefore, an incorrect use of terms to speak, as is often done, of " mathematical certainty " in relation to the results of ordinary reasoning, that is, reasoning not directly concerned with scientific or mathematical enquiries. In this usage, " mathematical certainty " must simply mean correctness of inference, or absence of fallacy in the reasoning process. A detective, for instance, may speak of the mathematical certainty of his conclusions when on the ground of a few clues, by means of a chain of strictly rigorous inferences he has satisfied himself of the guilt of his suspect.

It should, perhaps, be noted before proceeding, that although the more modern view of mathematics as the science concerned with deduction seems to cut loose the close ties with number and magnitude which it has always had in the more popular view, yet the notions of number and quantity must always

[1] This (v) is not accepted without modification by all thinkers.
[2] Cited by Stebbing, *Mod. Introduction to Logic*, p. 180.

remain prominent—the most prominent—topics of mathematics, since it must be with these notions that mathematics in its applications will be chiefly concerned.

Of itself, then, mathematics has nothing whatever to say about the physical world. It is concerned only with deduction. It may be applied to physical data. These must be the result of observation and are part—indeed the basis—of physics and the other sciences. Applied mathematics is in one sense indistinguishable from pure mathematics. Both are concerned with the making of deductions from given propositions. And, as Whitehead says: "When once the fixed conditions which any hypothetical group of entities are to satisfy have been precisely formulated, the deduction of the further propositions which will hold respecting them, can proceed in complete independence of the question as to whether or no any such group of entities can be found in the world of phenomena. . . . The difference (between "pure" and "applied" mathematics) is a difference of method. In "pure mathematics" the hypotheses which a set of entities are to satisfy are given and a group of interesting deductions are sought. In "applied mathematics" the "deductions" are given in the shape of the experimental evidence of natural science and the hypotheses from which the "deductions" can be deduced are sought. Accordingly every treatise upon applied mathematics, properly so called, is directed to the criticism of the "laws" from which the reasoning starts, or to a suggestion of results which experiment may hope to find. Thus if it calculates the result of some experiment, it is not the experimentalist's well-attested results which are on their trial, but the basis of calculation."[1]

The mathematician as such thus works in a kind of aloofness from the world of physical facts. "Here's to Pure Mathematics, and may it never be of use to anyone!" The toast, attributed to a mathematical don, illustrates finely the seclusion in which the pure mathematician carries out his work and erects his edifice of theorems. The cult of knowledge for the sake of knowledge was never so truly exhibited as in the upbuild of mathematical theory, which can be properly appraised and appreciated only by mathematicians themselves. For to few even of educated people is it given to enjoy the extent, the inventiveness, the beauty of form and the

[1] *Ency. Brit. XIV Art. Maths. Nature of*, pp. 85 *seq.*

depth—the poetry, it may be not inappropriately styled—of mathematical knowledge and especially that shown in its growth during the last century and a half. In the attempt to form a correct idea of modern mathematical development, any retained knowledge of mere "school mathematics" is probably misleading. How many, for instance, have more than a vague notion of the content and meaning of the non-Euclidean geometries, the many beautiful theorems of the theory of numbers or the theory of abstract groups—to mention random samples? It is true that engineers make application of some of the more superficial layers of mathematics, and that physicists from rather deeper strata cull parts they can use. It is also true that mathematical methods and ideas have during the last quarter century progressively infiltrated all the sciences, even such sciences as physiology, sociology, biology and psychology. It remains true, however, that in the higher reaches of pure mathematics no one but the mathematician himself is competent to survey the scope of the subject or completely to assess its worth.

The knowledge of this state of things, combined with the universally acknowledged power and utility of mathematics in its many applications, tends on the part of the mathematicians to a self-complacency of which the signs and portents are not lacking. The belittlement of common-sense, the arrogation to mathematics of a place of dominance in the gamut of the sciences, the attempt to set forth physical science as a purely deductive system, the minimization of the importance and the function of experimental methods—these are some of the indications that there is need of plain speaking about the place and status of mathematics in science. For while public ignorance provides a fruitful soil for the propagation of extravagant and fantastic ideas about mathematics in its relation to other branches of knowledge, and while it may be true that only a mathematician can fully estimate its worth and beauty, it is, notwithstanding, quite unnecessary to be a mathematical expert in order to understand its limitations. All that is needed for such purpose is a knowledge of its logical foundations—its nature.

It may first of all be observed, in passing, that the mathematician, *qua* mathematician, is a specialist. This qualifies him, if he is proficient, to make pronouncements about mathematics, but about nothing else. It is a well established psychological doctrine that we cannot learn one thing by doing another. Not only is it true, for instance, that we cannot make ourselves competent biologists by the study of physics, but it is just as certain that we cannot make ourselves exact thinkers in general by the study of mathematics in particular. Exemplifications of the truth of this principle are not far to seek. The characteristic feature of pure mathematics, and especially modern pure mathematics—say since the days of Cauchy (1789–1857) is its assiduous rejection in its constructive work, of all assumptions but its own explicitly recognized axioms and postulates. Mathematical certainty springs from this rigour and from nothing else (except of course the avoidance of formal fallacies). The successful pure mathematician is he who can think constructively while making no assumptions other than those stated as an integral part of the whole argument. And yet highly distinguished, even brilliant, mathematicians, in their general thinking, fail of these cautionary measures. Professor G. H. Hardy, for example, in his otherwise justly valued *Course of Pure Mathematics*[1] says, " It is stated in the Bible (1 Kings vii, 23, 2 Chron. iv, 2) that $\pi = 3$." This, it must be said, is a false statement about the Bible. But, more to the point, its falsity is a direct result of the infringement of the canons of mathematical thinking. For it rests on at least two implicit, untrue and indefensible assumptions. The first is that the Biblical writers cited were making statements about the relation between the circumference and the diameter of the circle ; and the second is that the measurements they gave were given as those of a plane circle. Both these assumptions are false and reprehensible. An unbiased reading and consideration of the Bible passages shows that the question of the relation of circumference to diameter is neither directly raised nor indirectly involved in them ; and that the dimensioned description is that of a vessel of circular cross-section indeed, but having a wall a hand-breadth thick and furnished with a double row of " knops " under its out-curving brim. These features made it inexpedient, if not impossible, to take the measurements of the diameter and the girth of the vessel at the same level. If the former were measured, as would be natural, by stretching a line across the mouth from brim to brim, and the latter by passing a line round the vessel at the most convenient place, *viz.* below the " knops," the ratio would as a matter of course be reduced from the known value of π to something

[1] 9th Edn. 1944, p. 70.

approximating to 3. And to make the girth exactly three times the diameter may well have been a designed feature of the sacred symbolism. Professor Hardy has not carried over into his general thinking the rigour so characteristic of his strictly mathematical thought.

Again, if anyone tends to the view that really eminent mathematicians—or, at least, mathematical physicists—are *always* clear thinkers, let him read that penetrating critique *Philosophy and the Physicists* by the late Professor L. Susan Stebbing. Dealing particularly with the popular scientific expositions of Jeans and Eddington, and more especially with their incursions into philosophy, this acute logician provides us with an arresting vindication of the rejection, by modern psychologists, of the old "faculty psychology" and of the related doctrine of "formal training," that is of the teaching which asserted that general "faculties," e.g. judgment, existed and could be trained by special practices, e.g. study of the classics. First paying genuine and high tribute to these writers as eminent in their own sphere as mathematical physicists, Professor Stebbing proceeds by merciless analysis to expose in their popular works their departures from the rules and habits which govern scientific thinking and scientific exposition. Their use of deliberate equivocation, their appeals to emotion, their comparisons of the disparate, their use of disguised assumptions, their failure to define terms with the resulting inconsistency in the use of the terms and the consequent breakdown of their reasonings—by drawing attention to these and other traits, Professor Stebbing demonstrates once again that a brilliant mathematician is *not* necessarily a brilliant thinker in general. "The mathematics," in other words *pace* Lord Francis Bacon, do not make "an exact man"—except when he is doing mathematics.[1]

Now the proper function of mathematics in its applications to science, as indicated earlier, is not always recognized. It is sometimes considered, for instance, that the "certainty" attaching to rigid mathematical process passes over to the physical theories to the development of which mathematics is applied. This is confused thinking. It is, indeed, highly delusive. Mathematics touches physical science in two ways: (1) it comes

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[1] The popular scientific expositions (e.g. *Universe of Light*) of that truly distinguished experimental physicist, the late Sir William Bragg, do, *per contra*, seem to be remarkably free from the faults specified in the text above. This is noteworthy.

in, in a rudimentary form, in scientific measurement, and (2) it works out deductively the consequences of any scientific hypothesis, that is, it " proves " what results must necessarily follow from the hypothesis. This does not prove the hypothesis but only tells us what to look for experimentally or observationally in order to do so. It is observation, with or without experiment that supplies the only kind of proof available. That it is necessary to state and insist upon this simple and elementary principle is amply evident. Not merely the man in the street but those who claim to be serious thinkers go astray here. Mr. Arnold Lunn, for instance, writes of Newton as having " an immense faith in mathematics as the final test of truth. 'The certainty of a mathematical demonstration' was indeed the only certainty which he recognized as absolute."[1] Lunn thus confounds demonstration (which is deductive) with " the final test of truth " (which is observational). As a classical, if elementary illustration of the proper place and function of mathematics in science, it may be worth while to consider the prediction by Adams and Leverrier of the position and orbit of Neptune.

We notice four stages :—

(1) *Observation.* Perturbations of Uranus from its orbit as calculated on the supposition that the only bodies influencing it are the known ones.

(2) *Hypothesis.* The supposition is made that an unknown planet is possibly disturbing Uranus.

(3) *Mathematical Development of Hypothesis.* This consists of the calculation of the position, mass, etc., of a new and hitherto unobserved planet adequate to account, by its gravitational " pull," for the observed irregularities.

(4) *Observation.* Telescopes are directed to the calculated position. The new planet is seen. The hypothesis is confirmed.

Clearly the " mathematical demonstration " appeared in step (3) but the " final test of truth " did not come till step (4), and was furnished by observation. The " certainty " of the mathematical step merely told what certainly the observers must look for *and find* if the hypothesis was to be regarded as true. The truth of the hypothesis was not established until the calculations

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[1] *Flight from Reason,* p. 37.

of mathematics were found to agree with actually observed facts. *In physical science mathematics does not furnish certainty as to the soundness of a theory.*

It should be added here, to avoid misunderstanding, that when such an able mathematical astronomer as Sir James Jeans writes, as he does[1] of " mathematical tests " of the various tidal theories of the origin of the planetary system, he is being brief rather than precise. His meaning is clear. The " mathematical tests " which were justifiably demanded of, though omitted from, the speculations he was examining, were really the mathematical development of those speculations—the development which, by tracing their necessary consequences, would have enabled them to be compared with the known facts of observation. The real *tests* are the comparisons with fact. The mathematical work merely makes this possible.

But more subtle and illusive are the attempts sometimes made to represent science as entirely mathematical. The pretence is that, to those well enough versed in mathematics, the laws of nature may become known without resort to experiment—that the competent mathematical scientist may cut himself clear of concrete facts and work entirely in symbols. Chief among those who have made such claims is the late Sir Arthur Eddington who asserts that the aim of science is to " construct a world which shall be symbolic of the world of commonplace experience."[2] It is to be doubted if many men of science would accept such a lofty and pan-mathematical statement of aim. Professor H. Dingle, says, for instance, that science is " the recording, augmentation and rational correlation of those elements of our experience which are actually or potentially common to all normal people."[3] Professor Niels Bohr's view is, " The task of science is both to extend the range of our experience and to reduce it to order."[4] Mr. Bertand Russell says, " Physics . . . however mathematical it may become depends throughout on observation and experiment, that is to say ultimately on sense perception. The physicist asserts that the mathematical symbols which he is employing can be used for the interpretation, colligation and

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[1] *Universe around us,* 4th edn., p. 244.

[2] *Nature of Physical World, Intro.* Any criticisms of this distinguished mathematician and scientist here offered must be understood to carry with them a tacit acknowledgment of his brilliance and merited distinction.

[3] *Science and Human Experience,* p. 14.

[4] *Atomic Theory and the Description of Nature,* p. 1.

prediction of sense-impressions. However abstract the work may become it never loses its relation to experience."[1] These pronouncements agree in assigning prime and ultimate importance in science to experience and not to mathematics. In contrast, Eddington's *aim* is to create a system which is postulational and deductive and therefore completely mathematical. To effect it he postulates "relata" and "relations," assigning to both the irreducible minimum of distinctive characteristics. These relata and relations thus characterized are his "building material." From them, he professes to derive, by purely logical or mathematical process, the properties of space and the field laws of physics (though not, it should be well noted, the laws governing the electrons, protons and other elements of atomic structure). The procedure is most highly abstract. The extreme tenuity of the physical significance of the notions employed makes the author's dexterous reasoning difficult to follow. What does become clear from expressions of misgiving and cloaked apology on his part is that the assigned building material is found inadequate, since the builder is compelled to introduce into the structure "specially prepared material" from other sources. He thus breaks the "rules of the game." Far from deriving the properties of his symbolic world by deduction from its original primitive elements *as postulated* the fact is that the would-be builder has, perforce, all along the line, to keep his eye on the well-known world of nature and from the observed facts of that world assign to his postulated elements such additional qualities as will *make* the "constructed" world resemble the world of nature. The justice of this criticism may be apprehended by a careful study of the appropriate sections of his *Mathematical Theory of Relativity* and *Nature of Physical World*.

> "Why *four* numbers (as monomarks of the primitive building elements)? We use four because it turns out that ultimately *the structure can be brought into better order that way*; but we do not know why this should be so. We have got so far as to understand that *if the relations insisted on a threefold or a fivefold ordering it would be much more difficult to build anything interesting out of them; but that is perhaps an insufficient excuse for the special assumption of fourfold order in the primitive material*. . . . There is no reason to

[1] *The Scientific Outlook*, pp. 115–8.

> deny that a *diversity of worlds could be built out of our postulated material.* But all except one of these worlds would be still-born. Our labour will be thrown away unless the world we have built is the one *which the mind chooses to vivify into a world of experience.* The only definition we can give of the aspect of the relations chosen for the criterion of likeness is that it is the aspect which will ultimately be concerned in the *getting into touch of mind with the physical world.* But that is beyond the province of physics."[1]

What the world-builder here finds regrettable and calling for apology is (*a*) that his postulational elements are inadequate and have to be reinforced by observation, and (*b*) that the process of deduction is embarrassingly over-productive of symbolic worlds so that selection has to come into play. But what calls loudly for apology (which is not forthcoming) is his naive " identification " of the " ten principal curvatures " of his symbolic world with energy, momentum and stress, the familiar factors of our real world, " which are the subject of the famous laws of conservation of energy and momentum." As Professor Stebbing points out [2] this has nothing to do with logic. It is a fresh and large-scale *assumption.* In making it the builder is taking for granted Maxwell's laws expressed in his equations, which have behind them Faraday's assiduous and skilful experimental work, and equally Newton's formulations backed by the observations of Galileo and others. He is completely abandoning his logical game. We may remind ourselves that

> " When once the fixed conditions which any hypothetical group of entities are to satisfy have been precisely formulated, the deduction of the further propositions which also will hold respecting them can proceed *in complete independence of the question as to whether or no any such group of entities can be found in the world of phenomena.*[3]

But Eddington does not like this " complete independence." His mathematical world, if not built for the express purpose, has at least a sinister bearing which we are about to notice. He wishes on the one hand to import into this flimsy mathematical world of symbols the solidity and certitude attaching to the world

[1] *Nat. Phys. World,* Ch. XI : my italics.
[2] *Philosophy and the Physicists* (p. 68).
[3] Whitehead, *loc. cit.* my italics.

of perceptual fact, and on the other hand to impart to the empirically discovered laws of science the mathematical character belonging only to his world of postulation and deduction. And here lies the mischief of this ruse. "Granting that the identification is correct," he argues, "*these laws are mathematical identities*. Violation of them is unthinkable."[1] But as we have seen, the mathematically obtained structure exists apart from all relation with the real world. If the mathematics shows that *its* laws are truisms, i.e. that they follow necessarily from the definitions of the primitive elements, this fact has nothing to do with the laws of existing science, which are simply formulations of observed recurrences and regularities in nature. These do not follow from any definitions. They follow from—they are inductions from—perceptual experience. In short, they are observations. The only part mathematics plays is in their formulation. This principle is of the greatest importance. It bears closely on the relation between the laws of science and the admissibility of "miracle" to rational thought. If the field laws are mathematical identities, their infringement is, as Eddington proclaims, not merely impossible but *unthinkable*, for they are a mere paraphrase, as it were, of the definitions of the constituent elements. This would rule out miracle—finally. But as we have maintained, these laws are nothing more—despite Eddington's attempted legerdemain—than the periodicities and regularities of nature which have been the subject first of observation and then of mathematical formulation. They are based on observation. But so are the records of the authentic miracles of Biblical theology. Believers in the latter may hold to their belief in complete and consistent rationality. Indeed the irrationalism is with the unbeliever. For the rational admissibility of competently observed and duly attested miracle is seen to be *inherent* in the logical make-up of inductive scientific method.

If further confirmation of the true part played by mathematics were needed it is amply supplied by the history of science. In the course of the actual development of scientific knowledge the process has *always* taken the form of an alternation—a blending—of observation with theory, whether strictly mathematical or otherwise. This is true both of the older classical discoveries and also of the more recent and much more highly theoretical

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[1] *loc. cit.*, p. 231 (Everyman Ed.) Italics in original.

advances associated with the well-known names of Einstein, Heisenberg, Schrodinger, Dirac, De Broglie, Born and the rest. Writing for instance of the General Theory of Relativity, a scientific development in which mathematics, notably the theory of tensors, played a prominent part, Professor Max Born says, "The new theory is a gigantic synthesis of a long chain of empirical results, not a spontaneous brain wave."[1] It was by experiment, moreover, that the theory was regarded as established when the deflection of light predicted by it came under actual observation. Prof. Born gives numerous similar examples.

> "It is often said that it was a metaphysical idea which led Heisenberg to the principle of matrix mechanics, and this statement is used by the believers in the power of pure reason as an example in their favour. Well, if you were to ask Heisenberg he would strongly oppose this view. As we worked together, I think I know what was going on in his mind. At that time we were all convinced that the new mechanics must be based on new concepts having only a loose connection with classical concepts as expresssed in Bohr's postulate of correspondence. *Heisenberg felt that quantities which had no direct relation to experience ought to be eliminated. He wished to found the new mechanics as directly as possible on experience.* If this is a 'metaphysical' principle, well, I cannot contradict. I only wish to say that *it is exactly the fundamental principle of modern science as a whole, which distinguishes it from scholasticism and dogmatic systems of philosophy.*"[2]

This writer closes a most valuable and authoritative survey of very recent science with the following: "My advice to those who wish to learn the art of scientific prophecy is not to rely on abstract reason but to decipher the secret language of Nature from Nature's documents, the facts of experience."[3]

When Mr. Bertrand Russell said that "mathematics is the subject in which we neither know what we are talking about nor whether what we say is true"[4] he was in no way disparaging mathematics but merely portraying its abstractness. In mathematics alone, of all the sciences, is abstraction complete;

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[1] *Experiment and Theory in Physics*, p. 14.
[2] *ib.* p. 18—italics mine.
[3] *ib.* p. 43.
[4] I regret that storage of my books prevents a precise reference to the source of this well-known dictum.

and it is abstract from beginning to end. From the moment when, in the kindergarten, the young child recognizes that notwithstanding the different pictures on them, two blocks here equal two blocks there, the degree of abstraction increases apace until a stage is reached in which the student may be able to discuss the doctrine of propositional functions or manipulate anti-commuting operators in the study of group structure. All the concepts of mathematics are entirely remote from actual experience. No one, for instance, has had experience of a geometrical point or a geometrical straight line. Further, the deductive systems of mathematics, as we have already seen, are such that they may never correspond to anything given in experience. There are many systems of geometry, for instance, founded on different sets of axioms and postulates. And of these one only can correspond to real space. (Which one actually does so correspond a cautious thinker would agree has yet to be finally decided.)

It is easy to see the immense power gained by this abstraction. All the utilitarian and cultural values of mathematics from simple counting to the theory of numbers and from simple mensuration to elliptic or hyperbolic geometry spring from it. What is not so often taken account of is the loss resulting from abstraction. Absurdly extravagant claims for mathematics are often made by those who lose sight of this. It is only in a very restricted sense—the numerical sense—of the word "equal" that two pictured blocks are equal to two other pictured blocks even of the same form and dimensions. At this very early kindergarten stage even, the child, in order to grasp the numerical concepts, has to abandon mentally the pretty and varying pictures and colours, the positions and orientations, the nearness or remoteness, roughness or smoothness, considerations of material, ownership, intrinsic worth and the like—in fact everything that is qualitatively, psychically, ontologically rich and varied. Our mathematical savants are too prone to leave this profound fact out of consideration. When it is said, for instance, that "all the pictures which science now draws of nature, and which alone seem capable of according with observational fact, are mathematical pictures,"[1] or again when the same writer suggests that "the Great Architect of the Universe now begins to appear as a Mathematician,"[2] he forgets that for the purposes appro-

[1] Sir James Jeans, *Mysterious Universe*, p. 127.
[2] *ib.* p. 167.

priate to mathematical physics he has been compelled to view the universe through the spectacles of complete abstraction. He has seen it only in its quantitative and relational aspects. It surely is not very intelligent or profound to ignore everything that is not mathematical and then to announce as a discovery that all one sees is mathematical. Of course our chemists, psychologists and biologists, not to mention our poets, artists and theologians, have their own views on this. What "picture," for instance, can mathematics offer us of the mysterious link between brain, which is physical, and thought, which is psychical —or of the laws which regulate the interaction between the two factors, so totally diverse yet so intimately bound together? What of colour? The influence of mathematics in science is such that we cannot investigate colour scientifically without *losing* colour in all its richness and deep psychic significance and finding ourselves alone with a bare number—a wave-length. Physics is not the whole of science and science is not the whole of thought. Neither is thought the whole of life.[1] The universe is richer in every way than can fall to the lot of a mere mathematician to imagine or a mathematical physicist to discover. There are multitudinous aspects of objective reality which cannot be brought into the domain of mathematics. These qualitative features which constitute so great a part of what may be called the ontological richness of nature, are an essential part of the domain of science over and above the merely metrical aspects; and, properly viewed, not a whit less important. They are, indeed, an indispensable check on the otherwise uncontrolled tyranny of numbers and of mathematical form. Further, as Dr. W. R. Thompson, F.R.S., demonstrates,[2] the metrical is itself dependent on them—the quantitative is dependent on the qualitative. For example, our perception of space is based on the perception of bodies; and our apprehension of the tridimensionality of bodies is one of form.

Nor is the loss by abstraction without its serious side even *within* the realm of the quantitative—a seriousness which calls for a firm retention of hold on reality and common sense and the placing of the feet firmly on the ground of experience. This necessity is capable of elementary illustration. As the tyro in algebra knows, the mathematical formulation of an everyday

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[1] See Lamont, *Christ and the World of Thought*, preface, p. v.
[2] *Science and Common Sense*, p. 105.

problem, say by means of a quadratic equation, may lead to numerical results which are inapplicable to the problem in hand. In fact, the numerical results furnished by the equations may be (*a*) directly applicable as solutions of the problem, (*b*) totally inadmissible (e.g. a negative number of children in a family) or (*c*) applicable as solutions not of the actual problem from which they arose but of a similar but modified problem (e.g. external, instead of internal section of a line-segment in a given ratio). Whence come, then, these inapplicable roots of equations? The equations give us *all* the cases in which certain numerical relations hold. The actual problem admits perhaps of only one of these as its solution. Why then does the actual problem bring us to the numerically more comprehensive equation? The answer is that in committing ourselves to the merely quantitative aspects we are committing ourselves to all the relations which may hold between the numbers expressing these quantitative aspects while at the same time abandoning the qualitative features of which the numerical relations take no account. These qualitative features if held in the mind would save us from error by serving as limitations and checks; but they find no entry into the equations. Whatever numbers the latter may bring us to, a man cannot have a negative number of children in his family, and a length of wire cannot be divided where it is not. Mathematics is formal. As the Aristotelian would have it, between the numbers and their relations on the one hand and the material facts of the problem on the other there exists the difference corresponding to that between formal causes and efficient causes. For complete soundness of mind, it would seem, we must attend to them both.

E. Cassirer says[1] "Every mathematical function represents a universal law which by virtue of the successive values which the variable can assume contains within itself all the particular cases for which it holds." This is not to be denied. But it is a wholly mathematical and therefore abstract truth. Commenting on it Miss Dorothy Emmet remarks, "This is plausible in mathematics because here we are *not concerned with empirical elements* but with the development of an idea defining a functional relationship."[2]

A further citation from Whitehead should sufficiently illuminate this point. "A complete existence is not a composition of

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[1] *Substance and Function*, p. 21.
[2] *Nature of Metaphysical Thinking*, p. 72, my italics.

mathematical formulæ, mere formulæ. It is a concrete composition of things illustrating formulæ. There is an interweaving of qualitative and quantitative elements. For example when a living body assimilates food, the fact cannot be merely that one mathematical formula assimilates another mathematical formula. . . . The fact is more than the formulæ illustrated."[1]

It is abundantly clear then that the very abstraction which endows mathematics with its great power and beauty imports into it the dangers accompanying all formal studies—the dangers and losses due to remoteness from reality. The only safeguard against these is constant reference to the real, the experiential —indeed to the observations, intuitions and reason of the ordinary person, that is to common sense. Mathematics and all true science transcend common sense. But if they at any time *offend* it, a good case can be made out for the view that it is the suggestions of the mathematician or the notions of the scientist that are in danger of departure from truth rather than those of common sense. A considerable treatise could be written on the meaning and importance of common sense. It is here used to denote that general intelligence with which we arrive at valid conclusions—the quality or endowment which is necessary to the ordinary citizen in the solution of life's problems, to the juryman in arriving at a true verdict, to the scientist (only in this case with the aid of technical refinements and already accumulated specialized knowledge) in forming inferences from his observations and experiments. The difference between the scientist and the man of mere common sense is one of elaboration of method and apparatus and of previously acquired specialized knowledge—of intellectual level rather than of intelligence. A further difference is that common sense is stimulated to action by the extraordinary, that is by a problem in life, whereas the intelligence of the scientist is stimulated even by the ordinary. "It requires a very unusual mind," says Whitehead,[2] "to undertake the analysis of the obvious." But, once set in action, common sense follows exactly the same general procedure as science. The stages have been excellently exhibited by Professor Stebbing,[3] who resolves into their elements the thought and activity of a man who reaches the door of his flat to find it bolted against him from within.

[1] *Adventures of Ideas* (Ch. IX).
[2] Cited by Stebbing, *Mod. Intro. Logic*, p. 235.
ib., pp. 233 *et seq.*

(1) *The observed fact* (the bolted door) sets up a problem. There is something to be "accounted for." The bolted door is an unusual feature in a complex situation.

(2) *An explanatory guess is made.* A burglar! This would *connect* the unexpected fact with other facts and make it fit in a complex of ordered fact.

(3) *Results of the attempt at explanation are worked out.* If a burglar, then he is either still within or has got out; but door is bolted on inside; exit from third floor window impossible; doubts exist at this stage of the validity of the "burglar hypothesis."

(4) *Inspection.* Forces the door and examines everything within. Burglar has left obvious traces and made his exit by parcels lift. Guess verified.

Comparison of these steps with those given earlier in this paper shows that the thought processes of science and common sense are exactly similar. The latter is the basis of science or as T. H. Huxley said, "Science is organized common sense." Nevertheless Sir James Jeans frequently writes in such manner as to identify common sense with obscurantism. He certainly implies that in adopting the doctrine of a spherical earth he abandoned his common sense.[1] Professor Lancelot Hogben also tells us, with considerable complacency, that he long ago gave up his belief in common sense.[2] It is preferable to say that in both these cases the common sense was retained, though plied with numerous additional facts of observation. The suspicion arises that this depreciation of common sense is undertaken in the interests of modern *doctrines of science which will not stand up to common sense scrutiny*. This does seem to be the case. It is well illustrated by the treatment accorded to time.

The concept of time, as Thompson makes clear,[3] is indefinable. (If anyone questions this let him produce a definition avoiding circularity.) It is indefinable because it is logically irreducible and it is logically irreducible because it is a primary intuitive concept. In itself it is a non-mensurable quantity, for we have

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[1] *See New Background of Science*, 2nd. edn., pp. 44–5, 118.
[2] *Nature of Living Matter.*
[3] *loc. cit.*, pp. 88–105. See the whole section to which the present writer is for much of the following argument greatly indebted. The whole book merits close study.

no direct means of equating units of it. It cannot therefore of its own right enter into mathematical formulæ or equations. It is only by virtue of its close relation to change, especially change of place, which *is* directly susceptible of quantitative expression, that it can be correlated with quantity. *Thus only indirectly can time be brought into the world of mathematical equations.* Moreover, time is irreversible. We may move backwards and forwards repeatedly through a point in space, but not through an instant of time. Now to employ time as one of four terms or variables in a "four-dimensional framework," as our mathematical physicists do, is to engage in the following procedure. Three symbols (the usual x, y, z of cartesian coordinates) are assigned for the three (Euclidean) spatial dimensions. A fourth symbol, t, is taken as representing time. In reality, this represents, not time, but a distance traversed by the hands of a clock; it is thus not truly a fourth dimension but one of the three ordinary spatial dimensions. It is, however, *called* a fourth dimension by analogy with the three real dimensions of space. This amounts to the substitution of an extra "dimension" for what is a real, intuitively apprehended, irreducible and irreversible flux, well-known as "time." There soon follow, with too great facility, (i) the treatment of t as positive or negative, (ii) the adoption of further symbols (e.g., p, q, r, etc.) which are also described by analogy as still further "dimensions" thus creating a "multi-dimensional" framework, and finally (iii) the embodiment of these in mathematical systems. What is the position, then, if such symbols have been built up into a system of equations from which the mathematician has obtained a set of numerical values identical with those of some natural process? *What is verified thereby? Not, most certainly not, the idea of time as dimensional or reversible; nor the physical existence of "space-time"; nor the physical reality of space of more than three dimensions.* For (as W. K. Clifford wrote in an analogous context) "whatever can be explained by the motion of a fluid can be equally well explained by the attraction of particles or the strains of a solid substance; the *very same mathematical calculations result from the three distinct hypotheses;* and science, though completely independent of all three, may yet choose one of them as serving to link together different trains of physical inquiry."[1] What

[1] Cited in Stebbing, *Mod. Int. Logic*, p. 208; italics mine.

then is verified ? Simply the power of the mathematical formulation to throw into pattern or link together, i.e. to colligate, the quantitative aspects only of the results of physical inquiry ; and nothing more. For, of course, we must not forget that in order to reach this form we have abandoned the qualitative facts which serve to distinguish one entity from another and one mode of action from another.

If we insist that the verification applies to the physical reality of time as one of the dimensions or as reversible, or to the physical actuality of quadri- or multidimensional space, we must, for equally cogent reasons, be prepared to believe that when certain equations were found to give numerical values corresponding to those of the experimentally determined properties of light, these equations proved both the actual existence of the ether and at the same time that it was, simultaneously, a mobile fluid, a nebula of discrete particles and an elastic solid. And we must for the same compelling reasons believe that a refractive index of $\sqrt{-1}$, or one explicitly involving it, is, or has a physical reality. For according to Professor Bouasse[1] " Fresnel produced a theory of total reflection by postulating a refractive index containing explicitly $\sqrt{-1}$. Following the same line, MacCullagh and Cauchy constructed a theory of metallic reflection. The formulæ deduced from this hypothesis—*perfectly and deliberately absurd*—are in accordance with the facts, which are facts of a scale large enough to make the result important. The formulæ may therefore be considered good formulæ, and, in the sense previously considered, *true* formulæ. Their goodness or truth is not, however, judged by the conformity between the basic hypothesis and physical reality, but merely by their capacity to produce numbers in reasonable agreement . . . with those obtained by the measurement of nature."

But our mathematico-physicists will not see this. Instead, they endeavour to foist upon us such quite unacceptable illustrations and justifications of these hypothetical physical " realities " as are scattered throughout their writings. Russell, Jeans, Eddington, J. B. S. Haldane and others are equally at fault here. Their attempts are worse than misleading. They are an affront to the very common sense to which they purport

[1] *Scientia*, p. 22, Vol. XXXIII, 1923, cited by Thompson, *loc. cit.*, p. 87. Italics in first italicized phrase mine.

to make appeal. Eddington's "non-Euclidean world," as seen in a polished brass door knob[1] simply shows how a polished Euclidean surface can give a distorted image, which is, however, itself of course Euclidean. Haldane's Riemannian universe[2] is described, as Thompson points out[3] in terms such as "up," "down," "around," "the other side of," etc., which come from the intuitions of reality and which therefore *cannot* enable us to envisage a world of radically different geometry. We must know, first of all, what *is* "up," what *is* "down," what *is* "the other side of" in the Riemannian world. Haldane's light-hearted optimism, then, does not carry us very far. Russell on relativity is no better when he imagines an escalator moving with the speed of light, which of course no material object ever did or could do.[4] As for Jeans's illustrations of "curved space"[5], he both attempts to help the reader to imagine curved space[6] and asserts the impossibility of imagining the illustration.[7] All such would-be illustrators ought to bear in mind in framing their illustrations, what they of course quite well know as fact, that it is not a departure from parallelism, perpendicularity or flatness that establishes a region as non-Euclidean (for all these features occur in objects in Euclidean space); it is therefore in the power of none of these features to illustrate, even, Riemannian or other non-Euclidean space. What is necessary is radically different ideas about what constitutes parallelism, etc. Therefore, a region of space marked by bending, non-rectilinearity, distortion, etc. canont even *begin* to illustrate a world of "curved space." All it can do is to stimulate the imagination to picture *something different*, but the world so pictured will always be and must always be pictured in imagery which is Euclidean—our imagination having no other material to build with. To *conceive*, by means of a set of mutually consistent definitions and postulates, a world of Riemannian or other extraordinary space is possible; to build mathematical systems based upon such conceptions and to embody them in scientific theories is equally possible, and may be useful. But to imagine such a world visually or to prove by any number of numerical "verifications" its real

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[1] *Space, Time and Gravitation*, p. 11.
[2] *Possible Worlds*, p. 261.
[3] *loc. cit.*, p. 74.
[4] *ABC of Relativity*, p. 36.
[5] See *New Background of Science*, 2nd edn., pp. 120–1, 136–9.
[6] p. 137.
[7] p. 139.

existence is a totally different matter. It is a conception which is uninterpretable in real terms or in images drawn from the real.

And yet the mathematical physicists are not content to leave the matter thus. Professor G. Castelnuovo[1] urges that " though the utility of the concept of space-time constitutes a sufficient justification of it, one must go further and say that it constitutes *as an object of sensory perception*, an essential element of relativity theory."[2] If relativity theory really depends on this sensory perception of space-time, the theory is hopelessly ruled out, even if in the restricted sense " true," i.e., useful in providing numerical values which correspond with those given by experiment. But why this insistence? It can only be because the mathematical physicists have an uncomfortable conviction that they have ejected the sensory and intuitive elements from their schemes, and feel compelled, in lieu of replacing them, somehow to fill their place, since it is admitted that science must begin with observation and must finally return to it. The elimination of the sensory, like the deprecation of experiment, must result in a false science, in that it makes inordinate claims for what is after all only one factor—and that a strictly formal one—in scientific process, to the detriment of the remaining factors. Common sense would, on the other hand, decry such a procedure from the outset.

To revert a little, the replacement of the intuitive concept of time by a dimension is the repudiation of the real nature of time, which is succession. It amounts, further, to the rejection of real causation, which is seen in succession, and which cannot be got into equations. This repudiation is, as we have seen, effected by a mere change of name. Time is called a dimension and treated as if it were such. Time thus treated as a dimension is really space, though called time. Of this process of the substitution of a " quantitative correlate " for an entity which is not quantitative and hence is " refractory to mathematical process," Thompson says in a pregnant passage, " *These homonyms of real things* undergo, in the mathematical world, certain transformations that are indeed consonant with their true natures of mathematical entities, though altogether repugnant to the natures of physical realities; but *the names they*

[1] As already cited *Trans. Vict. Inst.* 1943, p. 88.
[2] See Thompson *loc. cit.*, p. 92.

continue to bear create in the investigator the illusion that he is recasting the philosophy of nature. Thus we have particles without substance and waves that are not, for the mathematician, waves of anything, and indeed represent nothing more than an attempt to portray in a language that has evolved in a world of tangible realities the unsubstantial and indescribable figments of the mathematical universe. These myths or metaphors—translations of the untranslatable—have, however, been organized by the mathematical physicists, particularly the exponents of the Theory of Relativity into *an engine destructive of common sense,* which remains, as we have already said, the basis of the inductive sciences, as of normal thought."[1]

We live in an age of the easy acceptance of the unintelligible. There is a type of mind to which the very unintelligibility of a doctrine may commend it. But we should not accept the muddiness of a stream as evidence of its depth. "It is a safe rule to apply that, when a mathematical or philosophical author writes with a misty profundity, he is talking nonsense."[2]

The mathematicians, moreover, should put their own house in order before seeking to direct and to dominate in the world of science. There are serious and unresolved differences among them. They are not even in agreement about so fundamental a concept as number. While Russell makes it a generic concept arrived at by abstraction of the common elements of a group of objects, and so defines it as "the class of all classes equal to a given class," Cassirer makes it a relational concept, i.e., one derived from the notion of a relation between symbols in a certain form of serial order. It must be admitted that Cassirer's view has much to commend it. The difference may be one of those due to the difference between the analytic and the synthetic approach. Certainly number would never have been *reached* as a "class" concept, whatever may be revealed by analysis of the concepts, since "the individual groups must first be determined as ordered sequences of elements (i.e., by the ordinal theory, according to which what a number is depends on its place in the system)."[3] We cannot go into this. The difference remains. Also the acute differences between the thorough-going logicians, the formalists and the intuitionists are apparently accentuated

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[1] *loc. cit.*, p. 93, italics mine.

[2] Whitehead, *Introduction to Mathematics*, p. 227.

[3] See *Nature of Metaphysical Thinking*, Miss Dorothy Emmett, pp. 71 *et seq* for a readable discussion.

rather than resolved. The brilliant British mathematical philosopher F. B. Ramsey, who regrettably died (1930) at the early age of 26, had written in 1925 expressing the hope that the serious logical faults in *Principia Mathematica* which have caused its rejection by many and the desertion of its line of approach could be removed.[1] At the time of his death, he was, however, coming to agree that there are irremediable errors in its system.[2] If this is so, mathematics, although characteristically logical, is after all irreducible to logic and has a different kind of necessity whose nature is extremely obscure."

Leaving these uncertainties to the mathematicians to whom they belong, let us recapitulate.

Far from mathematics being primary and dominant in science, to assert that it is so is the reverse of the truth. There are vast tracts of the true domain of science outside mathematics, which is the farthest from reality of all the sciences. True science might well encompass a great advance if this were heeded and acted upon. The dominant *influence* of mathematics in recent science has been such that great loss has been suffered by the latter on account of the abstractness of the former, a rich variety of the qualitative aspects and relations of nature having been inevitably lost in the passage through the mathematical sieve. There is more than loss. Arising from the formality of mathematical methods there is danger of introduction of positive error if, by way of test, touch is not closely maintained with observable experience. Confusion exists in the minds of mathematicians between the forms of their descriptions and the real world itself. It is illustrated by their toying with the idea of the reversibility of time. But time is absolutely irreversible. The universality and the hypostatic character of the quantitative and mathematical in nature are only apparent. They are " idols of the cave "—a mere result of viewing the world mathematically and by no other method. Hence, to regard mathematics as a pointer to the spiritual nature of reality is dangerous and misleading. Mathematics plays its valuable part in (*a*) the " quantization " of observation and (*b*) the deductive development of scientific theory, in both serving to give precision. But a theory is scientific in the strict sense only if it admits of development and testing. A theory is unscientific not because it rests on unproved assumptions but because it rests on assumptions of such

[1] F. B. Ramsey, *Foundations of Mathematics*, p. 1.

[2] R. B. Braithwaite, *Cambridge Studies*, Ch. I, *Philosophy*, p. 20 (1933).

a kind that testing them is out of the question. Of such sort were the mythological assumptions invoked by the Egyptians to account for the phases of the moon. Equally mythical are the mathematical theories of "curved space" and the like. They are incapable of being tested except by the giving of numerical coincidences, which as we have seen is inconclusive. Further, these myths infringe the rules of scientific terminology; for scientific terms should be unique in their reference, unambiguous and precise. "Curved space" merely confuses. Things may be curved *in* space, but "curved space" is unthinkable, because of the nature of the notion of space—an abstraction from our intuition of form. If the mathematical formulæ and equations associated with the hypothesis of curved space are found *useful*, in that they give numerical results coinciding with experience, well and good. That is, however, no justification for the introduction of an absurd analogy, much less a proof of its physical reality. That such notions are absurd and offensive to common sense is proved by the illogicality and inconsistency of the attempts to justify and illustrate them. They are of a piece with the homonymic falsities which create "dimensions" gratuitously and represent time as a dimension.

Mathematics has been of great and acknowledged use in some of its applications. Where it has been of the greatest use, e.g., in celestial mechanics (the "perfect science") and in engineering, it has worked most closely, hand in hand, with observation, experiment and practical knowledge. And its success in such branches is due to the fact that in them quantitative factors are of the very essence of the subjects and so are necessarily prominent.

In short, there is no magic in mathematics. It is the science concerned with deductive reasoning, mainly occupied with number and quantity. If this reasoning is built up on assumptions unfounded in experience, we have pure mathematics, which exhibits much beauty and inventiveness in its theoretical constructs, but which is detached from experience in both its foundations and its results, and so is, from the practical standpoint, valueless. If the reasoning is, however, based on data which are the results of experience, we have applied mathematics. This is an entirely *dependent* subject, for not only its data but the necessary tests are also experimental. Mathematics does *not* hold the primacy. It is a valuable servant but a tyrannical and untrustworthy master.

Written Communications.

Brigadier N. M. McLeod wrote: At first sight the above paper looked formidable, not to say rather "high-brow," but when I saw the name E. H. Betts my fears vanished. Knowing the author's rare gift for rendering simple the most forbidding and complex argument I rather settled down to read and enjoy every word.

May I in support recall to mind the classic example of mathematical absurdity? I refer to the mathematical explanation of the *supposed "real" result* in the famous Michelson-Morley speed of light experiment, which consisted in proving, to the satisfaction of many of our leading scientists, that the ether of space consisted, not of a material medium capable of transmitting waves, but of mathematics, pure, but curved!

But, we may well ask: What becomes of this mathematical theory when it is known that the result of the experiment was *not a "real" one*, as had been assumed by certain scientists, against the conclusion of the experimenters themselves? For, not only did they, the experimenters, repudiate the false assumption, but a long series of further, much more elaborate and accurate, observations were carried out at Ether Rock, Mount Wilson, in all of which the ether stream was not only detected, but measured.* In the words of Prof. Piccard, "It vanishes as soon as the Michelson-Morley experiment comes within the scope of known physical effects"; or, in the words of Einstein and his colleague de Silter, during their stay at Mount Wilson in 1932: "We must conclude that at the present time it is possible to represent the facts without assuming a curvature of three dimensional space."† And yet there are people who still believe in the mathematical "curved-space continuum"!

Mr. E. W. Siddans wrote: I have found Mr. Betts' paper a very interesting and inspiring attempt to deal with difficult ideas.

He seems to take a rather dim view of the attitude of a "pure" mathematician (pp. 4 and 6) and to exalt the value of observational

* "The Ether Drift Experiment..." Reviews of Modern Physics. Vol. 5, No. 3. July, 1933.

† Proceedings of the National Academy of Science, Washington. 15th March, 1932.

tests and common sense in deciding "final truth" (p. 8). Yet (p. 14) he agrees that cautious thinkers are still not sure what system of Geometry "corresponds to real space." (What does "real" space mean ?)

The stress on what mathematics cannot deal with (p. 15) nor words explain (p. 20) is very good, but I doubt if the remarks about Time (pp. 18 and 19) and its non-availability for mathematical treatment are sound.

In mathematics, we conceive, say, length and the equating of units of it without worrying if the experiment can actually be performed—so why not treat Duration in the same way ?

It would have been good to read a concluding paragraph summarising the relationship which Mr. Betts would like to see between the faith of a Christian mathematician and his special subject. In particular, I should greatly value any suggestions which would show how a teacher of mathematics can present his subject so as to be a positive ally to those which more directly stimulate a Christian faith.

Mr. W. E. LESLIE wrote : I am not sure whether Mr. Betts accepts the Special and General Theories of Relativity. Will he please say clearly whether he does so ?

On page 19 he objects to expressions such as x, y, z. t. But if it is not proper to associate t with space co-ordinates it is not possible to give mathematical expression to motion—which sweeps away almost the whole of mathematical physics ! When Mr. Betts speaks of the distance traversed by the hand of a clock, does he mean a lady's wrist-watch, or Big Ben ?

In the early part of the paper we are given primary intuitions, logical inferences from those intuitions, experiment and observation as the tests of truth. But later the author adds to these very clear terms another test, "common sense." In so far as this includes intuition, logic, experiment, and observation, the use of a fresh term is unnecessary : in so far as it excludes them its value is very dubious.

Mr. LAURANCE D. FORD wrote : Mr. Betts' interesting paper on "The Use and Misuse of Mathematics" prompts the thought that mankind in its modern thinking has seriously erred. It appears that the same perverseness which in the field of Biology produces the fallacy of Evolution, in Pictorial Art gives birth to surrealism, cubism and the cult of the repulsive and ugly and, in Music, afflicts us with the cacophony of atonalism, has also extended to the hitherto unimpeachable regions of mathematics and produced such contradictions of thought as "*curved space*" and "*reversible time.*"

Have we arrived at a position with regard to the intellectual advancement of man analogous to that spoken of by Shakespeare, who says that we "ripe and ripe" and then that we "rot and rot"? There comes a time in furbishing a knife-edge, when you cease to make the blade any sharper, and begin to make it smaller.

As in all mental processes there is development, so, to all development (at least as far as man is concerned) there is a period somewhere or other. If he, by forcing things, will go the other mile, he finds his latest advances are no advances. He is like the infant sucking from its milk bottle after the milk has gone: he keeps up the motions of drawing fresh supplies, but they are only wind.

Have we come to the place in Scientific development of thought where we have reached dead centre (the zenith), and, refusing to accept the sentence of our limitations, press on and on, only to find we begin to descend the other side of the circle? And what lies before us then?

Somewhere or other, sometime or other, all researches must lead us either to God or the blackness of darkness of ignorance of all things—but God has made Himself accessible to man in Christ Jesus without scientific researches at all, or either the use or misuse of mathematics.

I am indebted to Mr. Betts for putting so plainly what I have often "*felt dimly.*"

Mr. C. S. GRANT wrote: In Chapter I of his *An Outline of Philosophy*, Bertrand Russell proposes not to define "philosophy," and proceeds to indicate those "problems and doubts" which beset philosophy and make Fausts of us all: "Alas, I have explored philosophy, and" etc., etc.* No definition of philosophy nor of a

* *Goethe's Faust.* Translated by John Anster.

science can be complete—so far as it can be made complete—unless it is definition by the subject matter itself. I have not (I think) lost my peace of mind, but with Faust: " And here I am . . . , No wiser than at first ! "

No branch of science nor of any learning can be seen in perspective if not seen in relation to all the rest of science and learning, and we know next to nothing of anything (if we would admit it), even when Sir James Jeans can write of the new quantum theory, " For it enables us—in principle at least—to predict every possible phenomenon of physics, and not one of its predictions has so far proved to be wrong. In a sense, then, we might say that theoretical physics has achieved the main purpose of its being, and nothing remains but to work out the details."*

" In a sense, ... nothing . . . . " ? Theoretical physics cannot be " seen " by looking at theoretical physics alone, any more than we can " see " an apple if we do not notice the tree and much else that too few would dream of imagining to be the very knowledge we are really after. How many pigeon-hole compartments of science and learning are there ? How many should there be ? Difficult would it be to possess knowledge in one piece ! Scarcely do I think that I am " with useless learning curst," but I have a deep sympathy for the restless Faust, in that arched, Gothic chamber.

To say that one equals one is not to open discussion on the profundities of mathematics, but though that little equation was familiar to the Babylonians and Egyptians long before the Christian era, neither could have told me why one equals one, and I still do not know why I should trust such an apparently simple-looking statement because I must, and because it has never let me or anybody else down, or because every little boy and girl would look at me if I dared mention these reflections to them.

The logical process of induction on the validity of which we must repose our faith if ever we are to trust a scientific law, *appears* to be no less well and truly founded than our faith in the simplest of equations. Faust perhaps thought it too terrible to contemplate the confusion and danger of lost faith should someone suddenly cast doubt upon the wisdom of the great scientists in their simple

* *Physics and Philosophy.*

faith. Undoubtedly, the necessity for such wisdom is the most awkward thing in the whole theory of knowledge.*

Science must place that faith in the vast edifice which it is building for itself, or it must perish. But the mystery about that stately edifice deepens. Sir Arthur Eddington has told us: "*Our present* conception of the physical world is hollow enough to hold almost anything . . . . A skeleton scheme of symbols proclaims its own hollowness." Italics mine. Again, "It can be—nay it cries out to be—filled with something that shall transform it from skeleton into substance, from plan into execution, from symbols into an interpretation of the symbols."† So perhaps I am not alone with my faith in believing that nature does not intend that our footfalls shall resound indefinitely as in an empty hall.

It might seem from the amount of evidence available, that mathematics might be better regarded as the science of ideas *par excellence*, in contrast to the outlook which looks at it—correctly from one aspect—as a "skeleton scheme of symbols." Pure or applied, mathematics without *ideas* is unthinkable, and if it were not, it would be as useless as the great bulk of philosophy which wrangles about the meaning of words, rather than go to the infinitely greater trouble of finding meanings and then words, inventing new words just as would be necessary.

Mathematics is termed an abstract science, but, pure or applied, if it is to be intelligible, it must deal with facts of experience, every time. If the philosopher does not think so, the mathematician in him is not worried. For 1,800 years the Greeks studied conic sections as an abstract science, but for 1,800 years they made calculations about *things*, of which they had knowledge by the senses. If I cannot define unity to my satisfaction, I do not hesitate to believe the idea is arrived at by the senses, in the same way as the idea of hunger. For me, one egg is one egg; shell and all. If I have two eggs, *one* egg is equal to *one* egg, no matter if one of them is bad, and I have abstracted nothing, any more than I have abstracted the roof of a house when I knock at the door. Sufficient for me that for the

* W. A. Sinclair (University of Edinburgh) has a philosophical theory which would explain why one should accept this "faith." Chap. IX, What is Truth? *An Introduction to Philosophy*.

† Epilogue, *New Pathways of Science*.

one egg I can produce the other. It is up to the other person what he does with the bad egg, if he gets it. The mathematical point is still there under the imaginary but powerful microscope, still as uneven to look at as a somewhat larger point under the less-expensive magnifying-glass. Otherwise—bad philosophy.

The only possible way to appreciate thoroughly the so-called Arabic system of notation (which evolved slowly, and is itself a structure of ideas) is by some means to be compelled to make do without it when making difficult, prolonged calculations and when inventing new mathematical *ideas*. The reputation of mathematics for difficult ideas in innocent-looking dress is not lost by describing a small circle and calling it zero. Descartes (not to overlook Fermat) fortunately prescribing an easy life, at least for himself, invented the method of co-ordinate geometry. Without Descartes, no Newton's *Principia*. Eureka! Ideas come anywhere—this time in the bath, perhaps because the idea of specific gravity is easier than its applications in school books. It is the idea which is important. Lastly, we have the invaluable idea of the variable; worth its weight in gold equal to the weight of ink wasted in the teaching of mathematics minus illumination by ideas.||

Ideas build up and lead to the independent discovery of the differential calculus by Newton and Leibnitz. Without Newton's ideas, Einstein could not have started a reign of ideas—in the boldest and most comprehensive fashion—the like of which has not been known before and which cannot yet be seen for its use in broadening and deepening the brooding spirit of man.

If undue emphasis appears to be placed upon (these somewhat disjointed) philosophical flights, only through its problems and doubts, no less than through its achievements, can the science of mathematics be seen as it "really is." In the task of understanding the problems and doubts is the great difficulty in a criticism or appraisement of mathematics. I regard as suspects the problems and doubts I have mentioned. They are too problematical and cumbersome, and somewhere so much in the way of philosophical thought that we are reminded of a bad, involved and unwieldy hypothesis which has required too much explanation to be convincing.

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* These examples illustrating the *ideas* in mathematics are expanded in *An Introduction to Mathematics* by A. N. Whitehead, to which I am indebted.

Mr. R. T. Lovelock wrote : The author is to be congratulated on a paper which presses home a lesson which is much needed in some quarters—that mathematics is but one of several useful tools which, by providing a form of mental shorthand, enable the mind to grasp a much larger selection of natural relationships than would otherwise lie within its power. Any effort to answer those who have come to worship it as some new and omnipotent deity capable of solving all problems in human life, and worthy of unquestioning awe and subservience, cannot but fulfil a useful purpose.

Perhaps the weakest point in the author's treatment, in which he lays himself open to question from the mathematician, is in his treatment of the relations between observation and calculation. Although not stated categorically, it is everywhere implied that "observation" has an element of "absolute verity" which calculation lacks. Compare his words : "But the final test of truth did not come till step (4), and was furnished by observation". Frequent quotation is made from Miss Dorothy Emmet's masterly work, *The Nature of Metaphysical Thinking*, yet the main lesson so consistently pressed home in that work is not very clearly brought out here—the lesson that what we frequently define as "truth" is but the symbolism of our mind in correlating our "percepts," and in relation to the absolute is every whit as much a "concept" as a mathematical function ; this point is also developed very ably by Karl Pearson in *The Grammar of Science*. St. Paul's warning that "The things which are seen are temporal ; but the things which are not seen are eternal" (2 Cor., iv, 18) is worth bearing in mind as an example of guidance by divine truth in an age of "Scientific Ignorance" which is still found unimpeachable before 20th Century Metaphysics.

The discovery of "Neptune" was an excellent example to quote when defining the inter-relation between observation and calculation, but the very real utility of mathematics, and its appreciable assistance without which many fundamental advances would have been impossible, seem to have been minimised in the attack which is made on Eddington's epistemological treatment of physics. It is possible that this slight injustice arises from the fact that quotation is drawn entirely from a "popular exposition" by that author, and his more serious works (although one of them is mentioned) are not cited.

Mathematics contain a symbolism without the use of which many of the more intricate physical relationships cannot be expressed *rigorously* and it follows that a popular exposition must lay itself open to detailed attack by a mathematician. A very good case may be made for the effort to explain these matters as well as possible to the layman, but when mathematics, as used by one of this country's leading mathematicians, is being criticised, it is surely fair to cite his *mathematical* works. In one place, for example, the author complains " that the process of deduction is embarrassingly over-productive of symbolic worlds so that selection has to come into play." If reference had been made to *The Mathematical Theory of Relativity* (development pp. 213–226, discussion pp. 226–228), the author would have found this matter dealt with quite frankly, and although only a selection of terms from the general tensor is chosen because they behave in accordance with *all* our percepts, it is pointed out that this is equivalent to saying that our five senses are only cognizant of phenomena which may be described by such a limited system. I feel certain that the author is fully persuaded that very many entities are existent in the universe, and form a spiritual world of which our senses are not cognizant. Had Eddington not found any excess terms in his general expression it would have been a legitimate criticism that he could not possibly be correct, having only described a portion of our ambient; that he has found too many for our perceptual experience does not prove that he has necessarily found a correct universal system (for such matters do not form the subject of a possible physical experiment), but since a correct tensor (if such exists) must essentially contain extra terms, it is unfair to criticise him on this count.

Again, the empirical method behind the amazing developments of Quantum Mechanics is eulogised. This development was analogous to the observations of Uranus which gave birth to the calculations of Neptune's orbit which later enabled it to be discovered. In complement to this, it is suggested that Eddington's posthumous work *Fundamental Theory* presents a mathematical analysis of the relationship between Relativity and Quantum Physics which will point the way to many new advances by the experimental physicist. The failure to acknowledge the very real assistance which Quantum Physics can derive from such analysis is a weakness in the present paper.

The subject of " time " is a very difficult and debatable matter and many writers have certainly been guilty (as the author says) of woolly thinking when they have failed to discriminate between physical time and the biological entity. The author is wrong, however, in implying that leading mathematicians claim any " absolute verity " for the curvature of space : in fact, Eddington, in *Fundamental Theory* shows that " space curvature " and " quantum uncertainty " are but two methods of taking into account the same phenomenon when calculating results, and implies that neither has any absolute significance (see also *Mathematical Theory of Relativity*, p. 197). The author is also guilty of an over-simplification when (on p. 22) he suggests that dimensional time encourages a rejection of causation. The doctrine of causation has probably done more in the last five centuries to undermine the fundamentals of " divinity " than any other weapon of the rationalist. It is surely one of the most hopeful signs of our age that we are beginning to realise that our normal use of causation is nothing more than the specification of sequence (*i.e.*, the description of distribution in time), and that when we seek for a " cause " in the absolute sense we only come to rest in " personality." The inner necessity which is felt by so many, of the need for a mechanical " causality " to explain experience rests primarily on the recognition of many phenomena outside of our discrete personality, and independent of other human personalities by which we are surrounded. When once we have recognised the existence of God as a " super-personality," all problems associated with the nebulous clash between miracle and natural law vanish ; the so-called law of cause-and-effect becomes a specification of sequence, with the divine personality as supreme and efficient cause, and certain local " perturbations " of this continuous field which we call human personalities. It is suggested that our modern blindness to this fundamental, and our persistence in thinking of the universe as only a machine was foreseen by St.Paul when he spoke of the last days when men should have " a form of Godliness," while they denied the power thereof (2 Tim., iii, 5).

Author's Reply.

Pressure on space demands that my replies to contributors to the discussion should be summary.

Brigadier N. M. McLeod provides an opportune illustration of the tendency to ignore or minimize experimental results. I thank him and Mr. L. D. Ford for their appreciative remarks. I fear that the answers to Mr. Ford's two questions must both be, yes. But I rejoice with him that God has, altogether gloriously, revealed Himself in Christ, in total independence of science or mathematics.

Several of my critics failed to read my paper with sufficient care. I nowhere criticized Eddington's mathematics, which has my unqualified admiration. It was his pan-mathematical scheme of science and his philosophy of the universe constructed thereon to which I objected. To have quoted from his more serious mathematical works would have been too technical for the VICTORIA INSTITUTE. Moreover, it was unnecessary, since Eddington was equally frank about the "excess terms" in the more popularly expressed extracts I actually cited. If Mr. Lovelock implied that the extra terms of the general tensor can in any way represent the entities of "a spiritual world of which our senses are not cognizant" I reject the idea as a daydream. We might equally well claim that the roots of quadratic equations which are inapplicable as solutions of particular concrete problems stand for spiritual entities beyond our ken. Surely we must distinguish between mental abstractions and spiritual realities. As to the indebtedness of both Relativity and Quantum Theory, as also their interrelations, to mathematics, why should I have stressed it? I incline to the suspicion that Relativity, Quantum Theory and "Fundamental Theory" have, all three, lost themselves in a mathematical maze—and that due to the homonymic treatment of time and dimensions to which Thompson so well directs attention (see citations).

I cannot agree that when the super-personality of God *plus* human personality has been allowed for there is nothing more left of causation than "distribution in time." See Thompson, *Sci. and Common Sense*, pp. 101–103, and also Stebbing, *Mod. Intro. Logic*, ch. xv, xvii and xviii. Causation is an intelligible concept necessary to and still used by science in spite of the evaporative influence of mathematics.

To Mr. Siddans I must point out (i) that the "rather dim view of the attitude of the pure mathematician" was due to the necessity of dealing with what mathematics *is* in its essential nature, rather than with its better-known activities and applications. (ii) I nowhere used the phrase " final truth " ; I did write of experiment as the final *test* of truth in the examination of a physical theory. To this I adhere. (iii) By " real space " I mean the space of experience and of experiment as when considering the volume of a flask, the expansion of an iron rail or the distance of Sirius. And surely, to leave it an open question which geometry corresponds to real space is merely to leave the decision to experiment and observation—consistently enough. (iv) To ask that duration be treated in the same way as length " without worrying if the experiment can be actually performed " is a sheer begging of the whole important question of what is the intrinsic nature of time and a source of serious error in philosophy if not in mathematics. Incidentally, the " experiment " of equating units of length *can* be (directly) performed. (v) Christianity cannot be got out of mathematics. I urge Mr. Siddans as a teacher to be wholeheartedly sincere, and to teach his mathematics with pointed regard to the distinction between primary assumptions and deduced results and with due recognition of its limitations as an organon of knowledge. Thus, he may inculcate habits of intellectual honesty and love of what is sincere and true. He can hardly do more. Christianity is a revelation, not a discovery.

It is really false for Mr. Lovelock to say that in my paper, " although not stated categorically, it is everywhere implied, that observation has an element of ' absolute verity ' which calculation lacks." I avoided with extreme care and, I believe, consistency, all reference to " absolute verity " or such ideas. It should have been clear enough to every reader from the mere context of the words " the final test of truth," that they referred to the finality only of the process of establishing (or refuting) any working hypothesis of science. I eschewed all approach to metaphysics. I nowhere referred to the " absolute " or even to the " real " except, in the latter case, with the simple meaning " belonging to every-day experience " or " subject to actual observation and experiment." For similar reasons, although pleased to quote (but twice only, not " frequently," as alleged) from Miss Dorothy Emmet's brilliant book, I did not feel called upon to summarize its " main lesson." For to say that " what we often define as ' truth ' is but the symbolism of our minds in correlating our percepts, and in relation to the absolute is every whit as much a ' concept ' as a mathematical function " may or may not be profound truth, but (although endorsed by Karl Pearson—an old teacher of mine), it makes not the least difference to the relationship of mathematics to experiment on the one hand or to spiritual realities on the other. The former, Mr. Lovelock admits, I correctly illustrated. Does he claim, with respect to the latter (spiritual realities) that they are to be equated to mathematical concepts, which he, with Eddington, *seems* to place on a higher spiritual level than non-mathematical ? I submit that the " unseen things " of 2 Cor. iv, 18, are spiritual entities of an order outside of and unapproachable by mathematics however refined, and known only by revelation and the work of the Spirit of God in regenerated minds (1 Cor. i, 20 ; ii, 14, etc.).

If it is true that leading mathematicians claim no " absolute verity " for space curvature, it *is* true that they claim " physical reality " for it. If not, why do they take such pains to enable us to imagine it in our minds ? Eddington says it is a " picture " but not an " hypothesis " (*Nature of Phys. W.*, p. 157). A picture of what ? And Castelnuovo says (*Scientia,* Vol. XXXIII, pp. 169–180) it must be regarded as an object of sensory perception and not merely as a useful concept. I submit again that " curvature of space " is an unscientific term since in its reference it is anything but unique, unambiguous or precise. It is an attempt—a pretence—to " translate the untranslatable."

Mr. Leslie asks me to say plainly whether I accept the Special and General Theories of Relativity. I answer that I accept them as mathematical formulations giving certain values agreeing with experimental results. This does not give them status as sound physical theories. Every mathematician knows that his equations may " work " without even symbolising real physical truth.

I did not leave commonsense undefined. Mr. Leslie, however, wishes on to me what amounts to his own definition of it—a definition by enumeration of ingredients which I cannot wholly accept. In any case, the general methodology of commonsense, as shewn in my paper, is that of science. But, over the latter, *as a test,* commonsense has the very distinct advantage of having its feet firmly planted on the ground of experience common to all. It is a watch-dog we should encourage.

Mr. Leslie has quite misread my remarks on time. Of course, time can be put into equations and so mathematical physics is not in danger of being "swept away." What must, however, be recognised if we are to think fundamentally and so with philosophic soundness, is that it does not enter such equations of its own right, but indirectly by virtue of its correlate, viz., space (whether space traversed by the hand of a wrist-watch or by that of Big Ben, of course makes no difference!). It may be treated, mathematically, as if it were another dimension. That does not make it one, nor does the numerical truth or correspondence to metrical facts of the equations in which it is so treated establish the physical truth of its assumed dimensionality or the reality of space-time curvature.

Mr. Grant's comments seem to be quite irrelevant. His definition of mathematics as the science of ideas is completely inadequate. It would not differentiate mathematics from any other branch of knowledge—not even from Herbartian psychology. To say that one equals one is by no means the same thing as to say that one egg equals one egg. The former, by the laws of thought, is necessarily true. The latter is not. Yet Mr. Grant doubts the former and asserts the latter.