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What Makes a Contradiction?

'There's only one thing impossible, Jack, an' that's for a chiel to pull his troosers on ower his heid.' Thus a rustic worthy is reputed to have defined the limits of speculation in a harbour-side argument in my home town of Wick. In more theological matters, it is curiously difficult to find any corresponding 'standard cases' of logical impossibility. Arguments in theology tend to be inconclusive. Opponents freely accuse one another of 'having it both ways', and the suspicion grows in the mind of the onlooker that no sharp criteria of contradiction are recognized in talk about God. Why should this be? If the One whom Christians worship is the God of truth, why is it apparently so difficult to pin down different beliefs about Him as clearly and sharply incompatible?

The object of this essay is to throw some light on these questions by taking a closer look at the way in which sharp contradictions come about – in particular, by showing how what we regard as a contradiction, in any field of discourse, depends on certain presuppositions concerning the subject-matter, which may often be held unconsciously by participants on both sides.

Contradiction in Mathematics

Most of us would take mathematics to be the most rigorous of argumentative disciplines, so let us begin by looking at the way in which mathematicians use the notion of contradiction. Suppose that in coordinate geometry we define two points P and Q, and give both of them the same coordinates (x,y). (This means simply that each lies at a distance of x units from the north-south axis and y units from the east-west axis.) Does this definition of P and Q *contradict* (i.e. rule out, as impossible) the statement 'P and Q are not at the same place'? The answer is of course *Yes*, if P and Q are defined as points *in the same plane*; but otherwise, *No*. Once we admit the possibility of a third dimension, the contradiction vanishes. It is perfectly possible for two points to have the same grid coordinates on a map, for example, without being at the same place. It simply means that one of them is vertically above the other, like aeroplanes 'stacked' before landing. One of the first things an airport radar operator learns is that two aircraft moving to the same position on the radar screen will not necessarily collide!

Simple though it is, we will find this an instructive example for our present purpose. A reader who *presupposed* that we were talking about two-dimensional geometry will probably have felt slightly cheated by our introduction of a third dimension. 'Oh, if *that's* what you mean, of course it's all right; but you didn't *say*', he may complain. For the sake of illustration, we may even imagine someone accusing us of having 'brought in the third dimension in order to escape from the self-contradiction inherent in our earlier statements'. How then should we answer?

In so far as pure mathematics can be likened to a game, a creation of our own minds, with clearly definable rules made by ourselves, I think we must agree that we gave our critics too little information to play the game properly. We ought perhaps to have prefaced our definition of P and Q with the words 'In a three-dimensional framework'. Equally, they must admit that they had no grounds for accusing us of self-contradiction until they knew *how many dimensions* the space of P and Q might have.

But another kind of answer is also possible. We could tell our critics that the information we gave them (P is at (x,y); Q is at (x,y); P and Q are not in the same place) was logically *equivalent* to informing them that the space in which P and Q exist *must have more than two dimensions*. If they had not been so eager to accuse us of contradicting ourselves, they could have learned something that they needed to know, by paying proper attention to the data. In the same way, the radar operator, having observed that the positions of aircraft have frequently coincided on his two-dimensional radar screen without any signs of a collision or a near miss, could in principle infer, without being told anything more, that they must be separated in a third dimension. *An apparent contradiction, both of whose terms are supported by experience, is the logical indicator of an unsuspected dimension.* Conversely, it is impossible conclusively to settle the question whether two statements about the real world are contradictory by appeal to logic alone. Proofs of contradiction are always *relative* to some assumption about the 'dimensionality' of the descriptive framework, in a generalized sense.

Sad to say, this proviso even questions the rustic example with which we began. In three-dimensional space, it is true that trousers can only be put on 'legs first'; but if four-dimensional geometry were applicable, geometers assure us that the body could be inserted into trousers (or removed therefrom) without passing through any of their apertures! If this is a little difficult to visualize, consider an analogous situation in two-dimensional space. An object inside a circular boundary can get outside (if confined to the plane of the circle) only by passing through the boundary; but in three-dimensional space, it can be removed by lifting it out of the plane of the circle, and *over* the boundary, without penetrating it. Hence there is no necessary contradiction in saying both (a) 'X has come from inside Y (or has entered Y) which has a closed boundary' and (b) 'X has not passed through the boundary of Y'. It all depends on the number of dimensions in which X is free to move; or we might equally well say that an additional dimension in which X is free to move is betokened by the fact that X has entered or left Y without passing through its boundary. In either formulation the arbiter is not logic, but brute fact.

Complementarity

Shocking though our mathematical examples may seem, the removal of apparent contradiction by admitting an extra dimension makes relatively modest demands on our imagination. It is, after all, only what our visual system does for us automatically with the discrepancies between the appearances of solid objects viewed by our left and right eyes. We see, not discrepancies, but *depth* in the field of view. In a clear sense, the two views are not contradictory but *complementary*.

In the recent history of physics, however, apparent contradictions of a still more shocking kind have had to be endured. When light rays or electric 'cathode rays' are sent through empty space, they are found to ripple round an obstacle, such as the bars of a grating, to form the kind of 'ripple pattern' that a system of *waves* would do, on a receiving screen at the far side of the obstacle. This has given rise to a highly successful 'wave theory' of light-in-motion and matter-in-motion, from which the ripple patterns produced on a receiving screen by passing such radiation through gratings of all different shapes can be successfully predicted.

On the other hand when such rays are emitted or absorbed, they behave equally unmistakably like a stream of *particles*, 'quanta' or 'bullets' of a definite fixed size. Worse still, when the emission is so weak that 'bullets' are given off and received only one at a time, the pattern eventually formed by the 'bullet holes' on the receiving screen or photographic plate is still exactly the same ripple pattern as when the emission was intense.

Now we come to the apparent contradiction. If only one bullet at a time is supposed to encounter the grating (and is not stopped by it), it must surely pass through only one or another of the apertures in the grating. Yet as the bullet holes accumulate on the receiving screen at the far side, the pattern they gradually delineate is found to be still the same ripple-pattern that would be expected if a *wave* had passed through the *whole* grating. Must we then conclude that each individual bullet has somehow gone through *all* the apertures at once?

Initially, this dilemma was felt to be so intolerable that physicists divided into those who accepted the 'wave' model and those who accepted the 'particle' one (or perhaps each on alternate days!); but in due course the weight of experimental evidence has forced us to recognize that *both* thought-models are valid and necessary to do justice to different aspects of the behaviour of radiation (whether optical or electrical). Gradually, it has come to be realized that the situations to which a 'wave' model applies are *not of the same kind* as those in which a 'particle' model is needed. 'Wave' models lead to correct expectations of the behaviour of matter-in-motion (or charge-in-motion, or light-in-motion, etc.) whereas 'particle' models correctly predict the behaviour of matter-on-impact (charge-on-impact, light-on-impact, etc.). Given this distinction, the two are never in practice contradictory, but *complementary*.

It is not my purpose, of course, to commend this state of physical theory as if it were in any way final. All it does is to illustrate dramatically, in terms of actual scientific history, a point which might otherwise seem artificial and academic: namely, that criteria of contradiction, however 'commonsensical', can be dangerously misleading when applied to descriptions of the real world. Always it is the facts, however bewildering, rather than argument, however plausible, that must have the last word. The man who would venture in the name of 'logic' to pronounce any physical event impossible has no guarantee whatever against the arrival of the event in question with a label attached: 'You didn't expect this, did you?'. Readiness to expand our descriptive frame in obedience to fresh data is in fact what is meant by the essential humility of science.

But we can learn something more from this physical example. Confronted with irrefutable evidence of wave-behaviour and particle-behaviour, the physicist is not content to abandon logic and cheerfully maintain a jumbled model with two incoherent ingredients. Instead, he asks at once *under which circumstances* each description is appropriate. Only when each has been *labelled* for the experimental standpoint from which it is known to be valid can he rest content. Similarly, nobody tries to check the plan and elevation drawings of a building for consistency by laying one on top of the other; but until each is labelled for the angle from which it claims to be valid, no one knows how to check whether the two can in fact be consistent as pictures of the same three-dimensional object. Even when different descriptions are known to be complementary, *identification of standpoint* remains a major task.

A third lesson can be learned from the case of wave-particle physics. If we had asked a nineteenth century physicist what hard evidence he had for his assertions about the nature of radiation, his answer might well have been in terms of *entities* such as waves, or particles, which he might claim were 'observable' under suitable conditions. A physicist today would (one hopes!) be more cautious. The hard evidence he would point to would not be observable *entities* but observable *events*, to which he might give hyphenated names such as 'electron-impact', 'photon-impact' and the like. It is in fact by tracing our data back to events, and patterns and probabilities of events, that we have discovered how to express the facts of atomic physics without any trace of self-contradiction. This is not (as some positivists would have it) a matter of *denying the reality* of the entities confronting us, but only a principle of 'conceptual hygiene' to allow our limited experimental knowledge enough room to grow without breeding spurious contradictions. The entities whose existence we intuit or read off from our data are thus left the more free to impose their true structure upon our thinking as our data (in observed events) accumulate.

Logic and Theology

We may sum up what we have learned from these examples as follows:

(i) In any field of discourse, logic can be used to detect contradiction only when the dimensionality of the descriptive frame has been fixed. Otherwise, every apparent contradiction must be qualified as 'conditional on the non-existence of yet another (logical) dimension in addition to those which we have assumed'.

(ii) In discourse that purports to describe reality, the number of dimensions necessary to do justice to the data of experience must be absolutely open to revision by those data. No event can be held *a priori* to be logically impossible, 'contradictory to fact' or the like. Such claims are strictly nonsensical.

(iii) Where complementary descriptions turn out to be required by the data of experience, it is essential to identify the logical standpoint from which each is defined, as careless mixing of elements valid for different standpoints can lead to confusion.

(iv) It is easiest to see the logical relationship between different data and to avoid spurious conflict if they are expressed in terms of experienced events rather than abstract entities.

What then of theological discourse? Can we justly affirm that 'logic does not apply to the things of the spirit', or even that there is spiritual benefit to be derived from believing both sides of a contradiction in the name of faith? Not at all. Logic applies, and must be scrupulously applied, to any systematic statement worth making, in theology as elsewhere. What goes wrong in religious disputation is not that the anti-religious are too logical, but that they are often not logical enough in discerning what possibilities are left open by the data. They are too eager to adopt and argue within an impoverished descriptive frame, rather than keep open the possibility that their intuitive frame needs additional dimensions. On the other hand, Christians also often fail to sort out and keep clear the logical standpoints from which different theological concepts are defined; and they could frequently avoid accusations of self-contradiction or 'meaningless' talk if they would take pains to frame theological evidence where possible in terms of events and activities (e.g. obeying-God, being-forgiven-by-God, being-guided-by-God, being rebuked-by-God) rather than entities (God, obedience, forgiveness, guidance, rebuke).

But if we reject the idea that theologians are privileged to defy logic, we must not undervalue the grain of truth that the assertion contains. What it usually means, in fact, is that in theology we have often to deal with concepts whose logical dimensionality is undefined. It follows, for the reasons we have considered, that logical criteria of contradiction can seldom be *applied* with certainty, and that prior notions of 'what seems reasonable' are liable (in the logical nature of the case) to be treacherous guides. These are logical facts, neglect of which has been one of the great weaknesses of theological 'liberalism' and 'rationalism' down through the ages; and nothing we have said should be allowed to diminish their force. Where the subject matter adequately defines its own dimensionality, logical conclusions can be drawn with full rigour; but the onus must be on the logician to prove that this is the case. This is precisely the difference between empirical discourse and an artificial language game.

On the other hand just as empirical science rests content, at a given stage of its development, with *working assumptions* as to the logical dimensions of its concepts, so theology in principle might hope to do the same, and in practice it often does. Within the limits of these working assumptions, potential contradictions can be identified and the discipline of logic pursued with full rigour, subject only to an equally rigorous obligation to make clear at each step what is being taken for granted. After all, it is to the living God of truth, and not to some abstract code of logical practice, that we are responsible for avoiding self-contradiction, and above all for avoiding self-contradiction in His name.